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Chapter 21

The Transcendental Aesthetic of Time

We measure time in its passing.

St. Augustine

§ 1. The Puzzle of Time

Unless you are a physicist, a philosopher, or a theologian it is probably safe to assume you do not expend a great deal of effort contemplating the nature of time. Yet from the classical Greeks right up to today there probably has been no more vexing and controversial question than "what is time?" Adler tells us,

> Though the idea of time is traditionally linked with that of space, it seems to be much more difficult to grasp. In addition to provoking opposite emotions from the poets, it seems to engage the philosophers in a dispute about its intelligibility. This goes deeper than conflicting definitions or analyses, such as occur in the discussion of both space and time. Whereas time seems no less clear than space to some thinkers, to others it is irremediably obscure. Struggling with what it is and how it exists, they are exasperated by its evanescence as an object of thought [ADLE: v3, 897].

Unlike the idea of 'space,' which tends to collect adjectives setting a context for the use of the term (e.g., phase space, metric space, solution space, etc.), the idea of 'time' has not been one that has attracted a great many descriptive adjectives. One witticism that has been used is, "time is what keeps everything from happening all at once." This is, of course, not at all useful as a definition but, perhaps, at least catches the flavor of what sort of topic we're dealing with when we talk about 'time.' Is it possible to define 'time'? Historically, attempts to do so have usually resulted in setting down two distinctions. At the most rudimentary level there has been a great deal of agreement in setting these distinctions, although as soon as one tries to get to more explicit descriptions of each there has not at all been universal agreement on what these two distinctions implicate nor what each distinction really means. The two distinctions are: 1) Time in the connotation of a measurement procedure; and 2) time in the sense of what-it-is that the first distinction 'measures.' The first connotation is invariably some idea of Relation and, as such, speaks of time in terms of *Existenz*. The second connotation often has involved a subreptive reification that makes 'time' into a thing-in-itself, a *Ding an sich*, but which at root aims at trying to understand time in terms of *Dasein*. Not surprisingly, it is the latter which is the historical battleground of greatest dispute.

Most scientists find little to take issue with in regard to the first connotation. Newton, for example, described the two-fold division of the idea of time in terms of an 'absolute time' (second connotation) and 'relative' or 'common' time (first connotation). Bergson used 'time' as a term more or less congruent with the first connotation and used the term 'pure duration' when speaking of the second. Many scientists adopt the more or less pragmatic position that it makes sense to speak of time in terms of the first connotation but there is no point in arguing about the definition of the second connotation. Feynman said,

> Let us consider what we mean by *time*. What *is* time? It would be nice if we could find a good definition of time. Webster defines "a time" as "a period," and the latter as "a time," which doesn't seem to be very useful. Perhaps we should say: "Time is what happens when nothing else happens." Which also doesn't get us very far. Maybe it is just as well if we face the fact that time is one of the things we probably cannot define (in the dictionary sense), and just say that it is what we already know it to be: it is how long we wait!
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> What really matters anyway is not how we *define* time, but how we measure it. One way of measuring time is to utilize something which happens over and over again in a regular fashion – something which is *periodic*. For example, a day. A day seems to happen over and over again. But when you begin to think about it, you might well ask: "Are days periodic; are they regular? Are all days the same length?" . . . It does seem . . . that days are about the same length *on the average*. Is there any way we can test whether days are the same length – either from one day to the next, or at least on the average? One way is to make a comparison with some other periodic phenomenon. Let us see how such a comparison might be made with an hour glass. With an hour glass, we can "create" a periodic occurrence if we have someone standing by it day and night to turn it over whenever the last grain of sand runs out . . . We now have some confidence that both the "hour" and the "day" have a regular periodicity . . . although we have not *proved* that either one is "really" periodic. Someone might question whether there might be some omnipotent being who would slow down the flow of sand every night and speed it up during the day. Our experiment does not, of course, give us the answer to this sort of question. All we can say is that we find that a regularity of one kind fits together with a regularity of another kind. We can just say that we base our *definition* of time on the repetition of some apparently periodic event [FEYN3: Chap. 5, 1-2].

This is perhaps a fine attitude for empirical science to take, but still the temptation to step away from it and speculate about time in the *Dasein* connotation seems to be irresistible. For example, the fantasy of 'time travel' has been around for quite some time, and all such speculations must necessary presume there is "sometime" to travel *in*. Every such speculation reifies time and clothes it in hand-me-down ideas taken from space's closet. The reification of 'space' and the Platonic speculations of 'geometrodynamics' that we discussed in Chapter 18 are applied to 'time' as well. Recently there has been more and more of a trend for at least a few scientists to argue seriously that 'time travel might be possible after all.'[1] The issue refuses to go away.

It is not hard to appreciate why this is. When we 'measure time' using whatsoever sort of

[1] e.g. Paul Davies, "How to build a time machine," *Scientific American*, vol. 287, no. 3, Sept., 2002, pp. 50-55.

clock mechanism, that which is characterized by the procedure is *Relation* and as such can implicate nothing more than the *Existenz* of time. However, an empty manifold is no manifold at all; the idea of *nexus* has no meaning without a matter of composition for which this *nexus* is the connection. When we speak of the existence of any object, it is possible to speak of a *Dasein* without a known *Existenz*. The noumenal *I* of transcendental apperception is an example; so is the undiscovered cause of an event. But we cannot speak with objective validity of an *Existenz* that has no *Dasein*. To do so is to claim to speak of the manner in which an absolute *nothing* exists, and this idea is self-contradictory. Margenau wrote,

> "Time is the independent variable in the laws of mechanics." This is probably the very best definition that can be given of time, and yet it does not define the idea completely. For whence do we get the laws of mechanics? Are they not formulated in conformity with an experience which already presumes time? These interrelations of knowledge, the web of postulates and observations encountered here require for their understanding a separation of those qualities of time which are epistemically rooted in Nature from others which are constructively established . . . All clocks establish operational rules of correspondence between sensed durations and conceptual time. But do they solve the problem, or are we still left with an ambiguity similar to that involved in the relation between measured coordinates and length? That depends on the reasons we have for believing that clocks maintain a constant rhythm . . . In the final stage, then, the mystery is pushed back into Newton's laws of motion: we *postulate* that bodies uninfluenced by forces move in such a way that equal distances spell equal times. We have, after all, not progressed very far beyond the Egyptians and their water clocks with respect to basic assumptions. There is no way in which purely constructional elements can be eliminated from conceptual time
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> Thus it becomes necessary for us to examine the postulational properties of space and time [MARG: 136-139].

These "postulational properties," as Margenau calls them, are the concepts that go into the making of ideas of what Margenau called 'conceptual time.' Many statements about 'conceptual time' usually end up being statements that concern the *Existenz* of time reified, and which therefore necessarily presuppose the *Dasein* of time-as-an-object, and hence speak of its *Existenz* through the notion of substance and accident. But what *Kraft* do we suppose attends the substance of this object? What, if anything, can influence it and what does it influence? Does time have a magnitude? The ability to measure 'time intervals' is one of the most accurate instrumentations possible to science today. But what is 'it' that this fabulously accurate instrumentation measures? And what, if anything, are we to suppose for properties of time at intervals smaller than the precision of these measurements can resolve?

Now it may perhaps already be obvious to the reader that as we pursue the idea of time-as-an-object we are going to plow headlong into serious and fundamental epistemological problems. All concepts of objects are formed from notions which are bound to the transcendental schemata of time-determination in inner sense. If, then, we treat time-as-an-object (which we must do if we are to develop a conceptual understanding of the pure intuition of subjective time), is it possible to do so without encountering a vicious circle in our reasoning? This is the Critical issue in the transcendental aesthetic of time. Let us briefly review some of the issues that have plagued and dogged the conceptualization of time.

§ 1.1 Augustine's Riddle

For St. Augustine the riddle of time was a crucial issue for how one was to properly interpret scripture, particularly the creation story in *Genesis*. Like Plato, the early Christian theologians held that time 'itself' was created by God. But this instantly posed a major theological problem from the opening line in *Genesis*: "In the beginning God created . . ." If 'time' was created "in the beginning" and God created it, doesn't this have to mean that God existed 'before' time existed? And isn't such an idea flatly self-contradictory? After all, if time 'began' it makes no sense at all, it would seem, to speak of anything 'before' time began. And if this idea is contradictory, then *Genesis* must be false; but, Augustine held, *Genesis* cannot be false . . . You can see the paradox Augustine faced. The problem rests squarely upon the question: What is time? Augustine took up this issue in *Confessions* XI.

> If an instant of time be conceived which cannot be divided into the smallest particles of moments, that alone is it which may be called the present. Which yet flies with such speed from future to past as not to be lengthened out with the least stay. For if it be, it is divided into past and future. The present has no space. Where then is the time which we may call long? Is it to come? Of it we do not say, "It is long" because it is not yet, so as to be long; but we say, "It will be long." When, therefore, will it be? . . .
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> What is now clear and plain is that neither things to come nor past are. Nor is it properly said, "There be three times: past, present, and to come"; yet perchance it might be properly said: "There be three times: a present of things past, a present of things present, and a present of things future." For these three do exist in some sort, in the soul, but otherwhere do I not see them: present of things past, memory; present of things present, sight; present of things future, expectation . . .
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> I said then even now, we measure times as they pass, in order to be able to say, this time is twice so much as that one; or this is just so much as that; and so of any other parts of time which be measurable. And if any should ask me, "How know you?" I might answer, "I know that we do measure, nor can we measure things that are not; and things past and to come are not." But time present how do we measure, seeing it has no space? It is measured while passing; but when it shall have passed, it is not measured for there will be nothing to be measured. But whence, by what way, and whither passes it while it is a measuring? Whence but from the future? whither, but into the past? From that, therefore, which is not yet, through that which has no space, into that which now is not. But what do we measure, if not time in some space? For we do not say single and double and triple and equal, or any other like way that we speak of time, except of spaces of times. In what space then do we measure time passing? In the future, whence it passes through? But what is not yet, we measure not. Or in the present, by which it passes? But no space, we do not measure. Or in the past, to which it passes? But neither do we measure that which now is not . . .
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> I heard once from a learned man that the motions of the sun, moon, and stars constituted time, and I assented not. For why should not the motions of all bodies rather be time? Or, if the lights of heaven should cease, and a potter's wheel run around, should there be no time by which we might measure those whirlings, and say that either it moved with equal pauses, or if it turned sometimes slower, otherwhiles quicker, that some rounds were longer, others shorter? Or, while we were

saying this, should we not also be speaking in time? [AUGU1: 94-96].

The basic argument Augustine makes here is one that the operational definitions of science mentioned earlier would have a hard time refuting if 'time' is an external thing. The past and the future are not 'here'; the past no longer 'really' exists and the future does not yet exist. And we cannot measure something that does not exist. Only the present exists *right now* but in another instant it will be the past and no longer exist (thus, 'has no space'). There is, so to speak, 'not enough time to measure the present.' And yet we *do* measure time. If we measure time, but if we cannot measure time if it is some external thing, then obviously time cannot be an external thing. What, then, is left for time to be? Augustine answers: Something in the mind.

> It is in you, my mind, that I measure times . . . [The] impressions, which things as they pass by cause in you, remains even when they are gone; this it is which, still present, I measure, not the things which pass by to make this impression. This I measure when I measure times. Either, then, this is time or I do not measure times . . . But how is that future diminished or consumed, which as yet is not? Or how that past increased, which is now no longer, save that in the mind which enacted this there be three things done? For it expects, it considers, it remembers; that so that which it expects, through that which it considers, passes into that which it remembers. Who therefore denies that things to come are not as yet? and yet there is in the mind an expectation of things to come. And who denies past things to be now no longer? and yet there is still in the mind a memory of things past. And who denies that the present time has no space, because it passes away in a moment? and yet our consideration continues, through which that which shall be present proceeds to become absent [AUGU1: 97-98].

Kant is often given credit for coming up with the idea that time is not something external but rather internal to man. We see here that Augustine came up with this idea and won this particular publication race by a margin of thirteen centuries.[2] Augustine characterizes time in terms of the threefold *modi* of "a present of things past, a present of things present, and a present of things future." He does, however, intimate time with perception and thinking, i.e. memory, sight, and expectation.

§ 1.2 Time and Motion

If one expects to find Aristotle dealing with the issue of time in his metaphysics (that is, the 'science of being *qua* being'), a surprise awaits. Time is barely mentioned in Aristotle's metaphysics. Instead we find his treatment of it in Books IV-V of his *Physics*. Aristotle begins his discussion of time with a discussion of the difficulties it presents.

> First, does it belong to the class of things that exist or to that of things that do not exist? Then, secondly, what is its nature? To start, then: the following considerations would make one suspect that it either does not exist at all or barely, and in the obscure way. One part of it has been and is

[2] Augustine's solution also solved his scripture interpretation problem. Because time is something in the mind of man, God stands outside it and there is now no paradox attending how He could create time.

> not, while the other is going to be and is not yet. Yet time – both infinite time and any time you like to take – is made up of these. One would naturally suppose that what is made up of things which do not exist could have no share in reality.
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> Further, if a divisible thing is to exist, it is necessary that, when it exists, all or some of its parts must exist. But of time, some parts have been, while others are going to be, and no part of it *is*, although it is divisible. For the 'now' is not a part: a part is a measure of the whole, which must be made up of parts. Time, on the other hand, is not held to be made up of 'nows' [ARIS6: 369-370 ($217^b30 – 218^a9$)].

Aristotle next examines whether 'time' is the same as 'motion' (κίνησις) or 'change' (μεταβολή). He concludes that they are intimately related, but are not the same.

> Now the change or motion of each thing is only *in* the thing which changes or *where* the thing itself which moves or changes may chance to be. But time is equally present everywhere and with all things. Again, change is always faster or slower, whereas time is not; for fast and slow are defined by time; fast is what moves much in a short time, slow what moves little in a long time; but time is not defined by time, by being either a certain amount or a certain kind of it. Clearly, then, it is not motion. We need not distinguish at present between motion and change.
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> But neither does time exist without change; for when the state of our minds does not change at all, or we have not noticed its changing, we do not think that time has elapsed . . . So, just as if 'now' were not different but one and the same there would not have been time, so too when its difference escapes our notice the interval does not seem to be time. If, then, the non-realization of the existence of time happens to us when we do not distinguish any change, but the mind seems to stay in one indivisible state, and when we perceive and distinguish we say time has elapsed, evidently time is not independent of motion and change. It is evident, then, that time is neither motion nor independent of motion [ARIS6: 371 (218^b11-219^a1)].

This seems to narrow it down somewhat. But are time and motion two different things that are indissolvably bound together, or is the one some kind of part of the other?

> Now we perceive movement and time together; for even when it is dark and we are not being affected through the body, if any motion takes place in the mind we at once suppose that some time has indeed elapsed; and not only that but also, when some time is thought to have passed, some motion also along with it seems to have taken place. Hence time is either motion or something that belongs to movement. Since, then, it is not motion, it must be the other.
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> But what is moved is moved from something to something, and all magnitude is continuous. Therefore the motion goes with the magnitude, and because the magnitude is continuous, the motion too is continuous, and if the motion then the time; for the time that has passed is always thought to be as great as the motion. The distinction between before and after holds primarily, then, in place; and where in virtue of relative position. Since then before and after hold in magnitude, they must also hold in motion, these corresponding to those. But also in time the distinction of before and after must hold; for time and motion always correspond with each other . . .
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> But we apprehend time only when we have marked motion, marking it by before and after; and it is only when we have perceived before and after in motion that we say time has elapsed. Now we mark them by judging that one thing is different from another, and that some third thing is intermediate to them. When we think of the extremes as different from the middle and the mind pronounces that the 'nows' are two, one before and one after, it is then that we say that there is time, and this that we say is time. For what is bounded by the 'now' is thought to be time – we may assume this.
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> When, therefore, we perceive the 'now' as one, and neither as before or after in a motion nor as the same element but in relation to a 'before' and an 'after', no time is thought to have elapsed, because there has been no motion either. On the other hand, when we do perceive a 'before' and an 'after' then we say that there is time. For time is just this – number of motion in respect of 'before'

> and 'after'.
> Hence time is not motion, but only motion in so far as it admits of enumeration. An indication of this: we discriminate the more or the less by number, but more or less movement by time. Time is then a kind of number. Number, we must note, is used in two ways – both of what is counted or countable and also of that with which we count. Time, then, is what is counted, not that with which we count; these are different kinds of things [ARIS6: 371-372 (219^a4-219^b9)].

Aristotle has almost closed in on his fundamental explanation of time. Time is "a kind of number." The obvious next question is: What kind? It does not seem to be an 'ordinary' kind of number, e.g. 1, 2, 3, etc. That kind of number is "that with which we count." Time, on the other hand, is "what is counted." But what does this mean? His distinction here seems to fall along modes that Kant would call *Existenz* (ordinary 'number') and *Dasein* ('that which is counted'). How and where do we come upon the *Dasein* of time? Aristotle finds this in the idea of 'now'.

> The 'now' in one sense is the same, in another it is not the same. In so far as it is in succession, it is different (which is just what its being now is supposed to mean), but its substratum is the same; for motion, as we said, goes with magnitude, and time, as we maintain, with motion. Similarly, then, there corresponds to the point the body which is carried along, and by which we are aware of the motion and of the before and after involved in it. This is an identical substratum . . . but it is different in definition . . . And the body which is carried along is different, in so far as it is at one time here and at another there. But the 'now' corresponds to the body that is carried along as time corresponds to the motion. For it is by means of the body that is carried along that we become aware of the before and after in the motion, and if we regard these as countable we get the 'now' . . . This is what is most knowable; for motion is known because of that which is moved, locomotion because of that which is carried. For what is carried is a 'this', the motion is not. Thus the 'now' in one sense is always the same, in another it is not the same; for this is true also of what is carried . . .
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> Time, then, is both made continuous by the 'now' and divided at it. For here too there is a correspondence with the locomotion and the moving body. For the motion or locomotion is made one by the thing which is moved because *it* is one – not because it is one in substratum . . . but because it is one in definition; for this determines the motion as 'before' and 'after'. Here, too, there is a correspondence with the point; for the point also both connects and terminates the length – it is the beginning of one and the end of another . . .
> Hence time is not number in the sense in which there is a number of the same point because it is beginning and end, but rather as the extremities of a line form a number, and not as the parts of the line do so . . . and further because obviously the 'now' is no *part* of time nor the section any part of the motion, any more than the points are part of the line – for it is two *lines* that are *parts* of one line.
> In so far then as the 'now' is a boundary, it is not time but an attribute of it; in so far as it numbers, it is number . . . It is clear, too, that time is not described as fast or slow, but as many or few and as long or short. For as continuous it is long or short and as a number many or few; but it is not fast or slow – any more than any number which we count is fast or slow . . . Not only do we measure the motion by the time, but also the time by the motion because they define each other. The time marks the motion, since it is its number, and the motion the time [ARIS6: 372-373 (219^b12-220^b16)].

What Aristotle is telling us here is made easier to understand if we appreciate the very different way in which the classical Greeks viewed the idea of 'number.' Today we think of numbers in terms of, e.g., the Arabic numerals. But to the Greeks, a line was a 'number', as was a triangle and a rectangle, etc. (Greek mathematics was geometry). If our modern day notation for music had been around back then, "♪" and "♫" would also have been 'numbers.' Number in the 'what is counted' sense has to do with order and connection rather than with mere 'counting.'

> And once more is this true in the case of music . . . because the musical harmonies, distessaron, diapente, and dispason are named for numbers; similarly all of their harmonic ratios are arithmetical ones, for the diatessaron is the ratio of 4:3, the diapente that of 3:2, and the diapason the double ratio; and the most perfect, the didiapason, is the quadruple ratio.[3]

With Aristotle we see the idea of 'time' as a 'timeline' take shape. A line is not a collection of points abutted side by side. The line is by definition continuous, and a 'piece' of a line is not a point but rather another line. 'Now,' on the other hand, is a 'point' – a boundary marker that divides a line but is not itself 'part of' the line. The picture he paints here of time as 'number' takes shape in the form of *ordinal* rather than cardinal numbers.

The ontological standing of time according to Aristotle is peculiar. Aristotelian time is not subjective, as it is for Augustine, because "time is equally present everywhere and with all things." Time is a kind of 'container' in which *kinesis* is said to be contained, as, e.g., when Aristotle says,

> Since the motion of anything must always occupy either an equal time or less or more time, and since, whereas a thing is slower if its motion occupies more time and of equal speed if its motion occupies an equal time, the quicker is neither of equal speed nor slower, it follows that the motion of the quicker can occupy neither an equal time nor more time. It can only be, then, that it occupies less time, and thus it is necessary that the quicker will pass over an equal magnitude too in less time.
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> And since every motion is in time and a motion may occupy any time, and the motion of everything that is in motion may be either quicker or slower, both quicker motion and slower motion may occupy any time; and this being so, it necessarily follows that time also is continuous. By continuous I mean that which is divisible into divisibles that are always divisible [ARIS6: 393 (232^b15-25)].

Aristotelian time is therefore somewhat analogous to Aristotelian 'place' inasmuch as it is viewed as a kind of boundary of motion. Time is not an Aristotelian 'substance' but rather is an Aristotelian quantity. Its corresponding Aristotelian categories are "When?" (*pote*)[4] and "How large?" (*poson* or 'quantity').

> Of quantities some are discrete, others continuous . . . Discrete are number[5] and language; continuous are lines, surfaces, bodies, and also, besides these, time and place [ARIS1: 8 (4^b20-25)].

We can say that Aristotelian 'time' is to 'motion' (*kinesis*) what Aristotelian 'place' is to 'body.' We have previously seen what an abstract idea Aristotelian place turns out to be. Time seems no less abstract in Aristotle's presentation of it. The idea that time is the 'number' of

3 Nicomachus of Gerasa, *Introduction to Arithmetic*.
4 Tradition later (mis)translated *pote* ('when?') as 'time'. 'Time' is χρόνος (*chronos*).
5 In the sense of "that with which we count": 1, 2, 3, etc.

motion comes across the centuries less easily in the form of Aristotle's 'time is what is counted' than does the idea of time as 'the number we count with,' i.e. time as the measure of motion. Twenty centuries later, we find that it is this idea of time which preoccupies the philosophers.

§ 1.3 Time and Duration

Newton's *Principia* and Locke's *Essay* were published within a few years of each other, and we have Locke's word that he had not read Newton prior to writing the *Essay*. We will see what Newton had to say about time in the next section. For Locke time is less at issue than is the idea of *duration*:

> 1. *Duration is fleeting extension*. There is another sort of distance, or length, the idea whereof we get not from the permanent parts of space, but from the fleeting and perpetually perishing parts of succession. This we call *duration*; the simple modes whereof are any different lengths of it whereof we have distinct ideas, as *hours, days, years,* etc., *time* and *eternity*.
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> 2. *Its idea from reflection on the train of our ideas.* The answer of a great man, to one who asked what time was: *Si non rogas intelligo*, (which amounts to this; The more I set myself to think about it, the less I understand it,) might perhaps persuade one that time, which reveals all other things, is itself not to be discovered. Duration, time, and eternity are, not without reason, thought to have something very abstruse in their nature. But however remote these may seem from our comprehension, yet if we trace them right to their originals, I doubt not but one of those sources of all our knowledge, viz. sensation and reflection, will be able to furnish us with these ideas, as clear and distinct as many others which are thought much less obscure; and we shall find that the idea of eternity itself is derived from the same common original with the rest of our ideas [LOCK: 155].

As we are about to see, Locke identified 'duration' as the essential 'thing' and time as its measurement. Here we should remind ourselves that, for Locke, our 'ideas' are the results of sensation and reflection, that the former is 'impressed' upon our minds from without (copy of reality), and that reflection combines 'simple ideas of sensation' and, therefore, gives rise likewise to ideas that represent composites of copies of reality.

> To understand *time* and *eternity* aright, we ought with attention to consider what idea it is we have of *duration*, and how we came by it. It is evident to anyone who will but observe what passes in his own mind, that there is a train of ideas which constantly succeed one another in his understanding as long as he is awake. Reflection on these appearances of several ideas one after another in our minds, is that which furnishes us with the idea of *succession*: and the distance between any parts of that succession, or between the appearance of any two ideas in our minds, is that we call *duration*. For while we are thinking, or whilst we receive successively several ideas in our minds, we know that we do exist; and so we call the existence, or the continuation of the existence of ourselves, or anything else, commensurate to the succession of any ideas in our minds, the duration of ourselves, or any such other thing co-existent with our thinking [LOCK: 155].

Locke goes on to set down his arguments for why 'duration' is the product of reflection upon the succession of Lockean ideas. By this he thinks to establish 'the idea of duration' as originating from nature, and why sleeping does not make a wreck of the empirical origins of duration. His serious theoretical departure from the ideas of Aristotle begins with his argument that succession (and therefore all that follows from this idea) does not come from *motion*. As we have already seen, time and motion are inseparable for Aristotle. Locke will change this, and by doing so will make duration have its own thing-like *Dasein*.

> 6. *The idea of succession not from motion*. Thus by reflecting on the appearing of various ideas one after another in our understandings, we get the notion of succession; which, if any one should think we did rather get from our observation of motion by the senses, he will perhaps be of my mind when he considers, that even motion produces in his mind an idea of succession no otherwise than as it produces there a continued train of distinguishable ideas. For a man looking upon a body really moving, perceives yet no motion at all unless that motion produces a constant train of successive ideas: v.g. a man becalmed at sea, out of sight of land, in a fair day, may look upon the sun, or sea, or ship, a whole hour together, and perceive no motion at all in either; though it be certain that two, and perhaps all of them, have moved during that time in a great way. But as soon as he perceives either of them to have changed distance with some other body, as soon as this motion produces any new idea in him, then he perceives that there has been motion. But wherever a man is, with all things at rest about him, without perceiving any motion at all, - if during this hour of quiet he has been thinking, he will perceive the various ideas of his own thoughts in his own mind, appearing one after another, and thereby observe and find succession where he could observe no motion [LOCK: 156].

The 'man becalmed at sea' example that Locke uses here is rather a weak argument since it is easily counter-argued that the man himself can move about the ship, that he is breathing all during this episode, and that, in short, he is never 'out of touch' with 'motion.' Locke probably thought so himself, for he follows this paragraph with a lengthy discussion of different ways that different 'motions' can go unperceived while succession is yet present in one's mind. In my opinion he fails to make airtight his case that there is ever any circumstance where 'no motion at all is to be perceived.' It would have been interesting to see him try to defend this thesis against Aristotle, and I know on whom I would bet as the victor in such a debate. But weak argument aside, there is one important subtlety that is missed in the *Essay*: When Aristotle says that time and 'motion' are indissolvably linked, 'motion' is *kinesis* – change of *any* kind. Aristotle would argue (and, in point of fact, *did* argue) that thinking is 'motion in the mind,' and from this point of argument Locke's 'idea of succession' *is* an 'idea of *kinesis*.'

Still, for all of this, Locke apparently at least convinced himself that "succession is not got from motion." When Locke says 'motion' he means the mechanical motion of 'extended bodies in space.' A case might well be made that Locke and Aristotle are not so far apart after all, but I think this would be a false comfort. The motion vs. *kinesis* issue is an issue because it has an ontological impact on how one regards 'time' or 'duration.' Let us see where this proposition took Locke.

> 16. *Ideas, however made, include no sense of motion*. Whether these several ideas in a man's mind be made by certain motions, I will not here dispute; but this I am sure, that they include no idea of

> motion in their appearance; and if a man had not the idea of motion otherwise, I think he would have none at all, which is enough to my present purpose; and sufficiently shows that the notice we take of the ideas of our own minds, appearing there one after another, is that which gives us the idea of succession and duration, without which we should have no such ideas at all. It is not then *motion* but the constant train of *ideas* in our minds whilst we are waking, that furnishes us with the idea of duration; whereof motion no otherwise gives us any perception than as it causes in our minds a constant succession of ideas, as I have before showed: and we have as clear an idea of succession and duration, by the train of other ideas succeeding one another in our minds, without the idea of any motion, as by the train of ideas caused by the uninterrupted sensible passage of distance between two bodies, which we have from motion; and therefore we should as well have the idea of duration were there no sense of motion at all.
>
> 17. *Time is duration set out by measures*. Having thus got the idea of duration, the next thing natural for the mind to do, is to get some *measures* of this common duration, whereby it might judge of its different lengths, and consider the distinct order wherein several things exist; without which a great part of our knowledge would be confused, and a great part of history be rendered very useless. The consideration of duration, as set out by certain periods, and marked by certain measures or epochs, is that, I think, which most properly we call *time*.
>
> 18. *A good measure of time must divide its whole duration into equal periods*. In the measuring of extension there is nothing more required but the application of the standard or measure we make use of to the thing of whose extension we would be informed. But in the measuring of duration this cannot be done, because no two different parts of succession can be put together to measure one another. And nothing being a measure of duration but duration, as nothing is of extension but extension, we cannot keep by us any standing, unvarying measure of duration, which consists in a constant fleeting succession, as we can of certain lengths of extension, as inches, feet, yards, etc. marked out in permanent parcels of matter. Nothing then could serve well for a convenient measure of time, but what has divided the whole length of its duration into apparently equal portions by constantly repeated periods. What portions of duration are not distinguished, or considered as distinguished and measured, by such periods, come not so properly under the notion of time; as appears by such phrases as these, viz. "Before all time," and "When time shall be no more" [LOCK: 157-158].

And here we have it: Our modern day distinction between time as *Dasein* ('duration') and time as *Existenz* (Locke's 'time'). Clocks 'measure duration' by providing periodically-repeating sensible events whose coincidence can be set against other events and thereby provide a 'time reference' for the latter. Although many modern scientists might take issue with Locke's idea of 'duration,' few scientists (and *no* experimentalists) dispute Locke's thesis that time is what we measure by clocks. I doubt if Feynman ever read Locke, but it is easy to see by comparison how congruent are Locke's statements here and Feynman's comment quoted at the beginning of this chapter. Locke goes on to describe why periodicity is desirable as a measure of duration and the role that 'the revolutions of the sun and moon' have played in mankind's history of 'measuring duration.' The most conservative physicist would have no problem with what he has to say in this regard.

There enters in at this point, however, two consequences of significant ontological importance with respect to the idea of time. One is a philosophical issue that Locke somewhat fails to note, the other a practical issue that he does discuss. The philosophical issue is this: To measure 'time' by means of the periodic motion of some clock means that there is a smallest interval of time (one period of the clock) that can be measured. A true and pure empiricist must tell us that nothing we can say about time has any sure meaning if we cannot measure the property of time in question. This includes the question of whether or not 'duration' is *continuous*. To say of duration ('time *per se*') "it is continuous" is, from the viewpoint of an experimentalist, a speculation. It has historically proven to be a very useful speculation; mathematical physics would be enormously more difficult without this assumption. But it is still, from the pure empiricist's viewpoint, a speculation. Indeed, in the latter half of the twentieth century the speculative character of the 'continuity of time' has slowly come more and more into focus, and some of the world's leading physicists have indeed dared to wonder out loud whether we should not raise the question of whether 'time' might not be quantized rather than continuous. One thing, though, is certain: Objective time (i.e. the 'time measured by clocks') can only be made known to us through measurements in integer numbers of some fundamental period. This is not because 'duration itself' is quantized; this we have no way of knowing with certainty because 'duration' (the *Dasein* of objective time) is a *noumenon*. It is because our measurements of it are quantized.

Locke appears to have had no particular qualms about presuming 'duration' to be continuous. He did, however, point out the importance of the fact that our knowledge of 'durations' can never be known with exacting certainty because we cannot know our clocks to be absolutely accurate, nor can we know with certainty that one period of a clock is exactly equal to another period of this same clock.

> 21. *No two parts of duration can be certainly known to be equal*. But perhaps it will be said . . . how could it ever be known that such periods were equal? To which I answer, – the equality of any other returning appearances might be known by the same way that that of days was known, or presumed to be so at first; which was only by judging of them by the train of ideas which had passed into men's minds in the intervals; by which train of ideas discovering inequality in the natural days, but none in the artificial days, the artificial days, or νυχθήμερα[6], were guessed to be equal, which was sufficient to make them serve for a measure . . . These yet, by their presumed and apparent equality, serve as well to reckon time by (though not to measure the parts of duration exactly) as if they could be proved to be exactly equal. We must, therefore, carefully distinguish betwixt duration itself and the measures we use to judge of its length. Duration, in itself, is to be considered as going on in one constant, equal, uniform course: but none of the measures of it which we make use can be *known* to do so, nor can we be assured that their assigned parts or periods are equal in duration one to another; for two successive lengths of duration, however measured, can never be demonstrated to be equal . . . All that we can do for a measure of time is, to take such as have continual successive appearances of seemingly equidistant periods, of which seeming equality we have no measure, but such as the train of our own ideas have lodged in our memories, with the concurrence of other *probable* reasons, to persuade us of their equality [LOCK: 159].

This utter unknowability of the apparently equal periods of mechanisms used as clocks presages

[6] From νυχθήμερον, a day and a night, the space of 24 hours.

Feynman's same statement over two centuries later. Note that here Locke, and Feynman as well, aren't arguing that clocks 'might not keep the same time all the time.' We all know this happens. They are arguing that even when a clock appears to be keeping perfect time, we can never know for certain that each of its periods is precisely equal to the others. There is no *experiment* we can do to prove an apparent equality is a 'real' equality beyond all doubt. Time is a parameter in the dynamical laws of physics, thus is made part of those laws, and these laws, being self-consistent, cannot be turned around and used to test non-constancy in a reference clock's time intervals.

This may, in the case of Locke, appear to be a strange position for one who takes the copy-of-reality hypothesis as a given. After all, if we know duration through reflection upon simple Lockean ideas, and if all such ideas originate in a *tabula rasa* of the mind, why should we not think that the 'impression' of equality in the periodic motions of a clock 'reflects' actual and real equality of periodicity? Locke's answer stems from his earlier conclusion that the idea of succession does not come from motion.

> 22. *Time not the measure of motion*. One thing seems strange to me, – that whilst all men manifestly measured time by the motion of the great and visible bodies of the world, time yet should be defined to be the "measure of motion": whereas it is obvious to everyone who reflects ever so little on it, that to measure motion, space is as necessary to be considered as time; and those who look a little farther will find also the bulk of the thing moved necessary to be taken into the computation, by any one who will estimate or measure motion so as to judge right of it. Nor indeed does motion any otherwise conduce to the measuring of duration, than as it constantly brings about the return of certain sensible ideas, in seeming equidistant periods. For if the motion of the sun were as unequal as of a ship driven by unsteady winds . . . or if, being constantly equally swift, it yet was not circular and produced not the same appearances, – it would not at all help us to measure time, any more than the seeming unequal motion of a comet does [LOCK: 159].

It is, in other words, repetition in succession and not motion that is judged when we measure time.

Not surprisingly, Leibniz disagreed with most of Locke's major arguments, although the counter-arguments he set down in his *New Essay* seem to me to be weaker ones than those of his opponent. The debate Leibniz hoped to engage in with Locke never came about, and so we do not know which man's views would have prevailed in the exchange of ideas. Leibniz, for example, did not agree that it was not 'motion' that "furnishes us with the idea of duration":

> A train of perceptions arouses the idea of duration in us, but it does not create it. Our perceptions never provide a sufficiently constant and regular train to correspond to the passage of time, which is a simple and uniform continuum like a straight line. Changes in our perceptions prompt us to think of time, and we measure it by means of uniform changes. But even if nothing in nature were uniform, time could still be determined just as place could still be determined even if there were no fixed and motionless bodies. Knowing the rules governing non-uniform motions, we can always bring them back to comprehensible uniform motions, and by this means predict what will happen through the various motions in combination. In this sense time is the measure of motions, i.e. uniform motion is the measure of non-uniform motion [LEIB1a: 152].

Leibniz obviously does not favor us in this statement with arguments that show the support of the various pronouncements he makes here. Possibly he was thinking about Kepler's work on the elliptical orbits of the planets. Unlike Locke, Leibniz was a first-rate mathematician and, as a rationalist, held that our knowledge of duration was innate to the mind.

> I hold that time, extension, motion, and in general all forms of continuity as dealt with in mathematics, are only ideal things; that is to say that, just like numbers, they express possibilities . . . But to speak more accurately, extension is the order of *possible coexistences*, just as time is the order of *inconsistent* but nevertheless connected *possibilities*, such that these orders relate not only to what is actual, but also to what could be put in its place, just as numbers are indifferent to whatever may be being counted. Yet in nature there are no perfectly uniform changes such as are required by the idea of movement which mathematics gives us, any more than there are actual shapes which exactly correspond to those which geometry tells us about. Nevertheless, the actual phenomena of nature are ordered, and must be so, in such a way that nothing ever happens in which the law of continuity . . . or any of the other most exact mathematical rules, is ever broken. Far from it: for things could only ever be made intelligible by these rules, which alone are capable – along with those of *harmony* or of perfection, which the true metaphysics provides – of giving us insight into the reasons and intentions of the author of all things [LEIB9: 252-253].

Leibniz also took issue with the idea that we cannot be certain 'two parts of duration' are really equal. His main point, however, turned on the idea that inequality can be detected and, indeed, that use of the pendulum had revealed that the time from noon to noon was in fact not equal from one day to another. Here, however, I think Leibniz missed the point, which is the problem of knowing for certain that two periods *of the same clock* that appear to be equal *are* equal. Locke never said we cannot compare one clock to another clock and pronounce whether they equal or unequal in period.

On the question of whether or not time was the measure of motion, Leibniz seems to come out ahead. He gently chides Locke for putting words in Aristotle's mouth.

> I have just explained how that should be understood. In fact, Aristotle said that time is the 'number of motion', not its measure. Indeed we could say that a duration is known by the number of equal periodic motions, each beginning when the preceding one ceases, for instance by so many revolutions of the earth or the stars [LEIB1a: 153].

This is an important point to make (bearing in mind that Aristotle did not separate 'duration' from 'time'). For Aristotle, time is 'what is counted' when we put a number to motion, not the number itself. The specific number laid against motion (e.g. '3 seconds') is the measure of motion, but time *per se* is not the same as the measure.

> *Number in the broad sense* – comprising fractions, irrationals, transcendental numbers and everything which can be found between two whole numbers – is analogous[7] to a line, and does not

[7] Remnant and Bennett point out that by 'analogous' Leibniz here means 'proportional' in the contemporary sense of 'ordinally similar' ('ordinal numbering' as opposed to 'cardinal numbering').

> admit of a minimum any more than the continuum does. So this definition of 'number' as the multitude of units is appropriate only for whole numbers. Precise distinctions amongst ideas of extension do not depend upon size: for we cannot distinctly recognize sizes without having recourse to whole numbers, or to numbers which are known through whole ones; and so, where distinct knowledge of size is sought, we must leave continuous quantity and have recourse to discrete quantity. So if one does not use numbers, one can distinguish amongst the modifications of extension only through *shape* – taking that word broadly enough to cover everything which prevents two extended things from being similar to each other [LEIB1a: 156].

This is, of course, a basic distinction between extensive and intensive magnitude. The 'measure of motion' is invariably a number in the 'whole numbers' sense, i.e. so many 'units of time'; but 'time' in the Aristotelian sense of 'number of motion' relates to motion in terms of an intensive ('ordinal') relationship. The distinction noted here is a key factor in Bergson's idea of 'pure duration' (which we quoted in Chapter 8 §6.4). Of duration and time, Bergson tells us

> Pure duration, that which consciousness perceives, must thus be reckoned among the so-called intensive magnitudes, if intensities can be called magnitudes: strictly speaking, however, it is not a quantity, and as soon as we try to measure it, we unwittingly replace it by a space . . . Granted that inner duration, perceived by consciousness, is nothing else but the melting of states of consciousness into one another, and the gradual growth of the ego, it will be said, notwithstanding, that the time which the astronomer introduces into his formulae, the time which our clocks divide into equal portions, this time, at least, is something different: it must be a measurable and therefore homogeneous magnitude – It is nothing of the sort, however, and a close examination will dispel this last illusion . . . Within myself a process of organization or interpenetration of conscious states is going on, which constitutes true duration. It is because I *endure* in this way that I picture to myself what I call the past oscillations of the pendulum at the same time as I perceive the present oscillation . . . Thus, within our ego, there is succession without mutual externality; outside the ego, in pure space, mutual externality without succession . . . Owing to the fact that our consciousness has organized them as a whole in memory, they are first preserved and afterwards disposed in a series: in a word, we create for them a fourth dimension of space, which we call homogeneous time, and which enables the movement of the pendulum, although taking place at one spot, to be continually set in juxtaposition to itself. Now, if we try to determine the exact part played by the real and the imaginary in this very complex process, this is what we find. There is a real space, without duration, in which phenomena appear and disappear simultaneously with our states of consciousness. There is a real duration, the heterogeneous moments of which permeate each other; each moment, however, can be brought into relation with a state of the external world which is contemporaneous with it, and can be separated from the other moments in consequence of this very process. The comparison of these two realities gives rise to a symbolic representation of duration, derived from space. Duration thus assumes the illusory form of a homogeneous medium, and the connecting link between these two terms, space and duration, is simultaneity, which might be defined as the intersection of space and time [BERG1: 106-110].

By the end of this chapter we shall see that here Bergson has laid his hand on the shoulder of the *Realerklärung* of 'time.'

§ 1.4 Newton, Einstein and Objective Time

We now come to the contemporary view of 'time' that dominates physics and yet, at present, is also the nucleus of ontological issues that have recently been receiving gradually more and more recognition. We will start with physics' definition of 'time,' as set down in the 4th (1999) edition of the *Oxford Dictionary of Physics*, and the commentary that accompanies it:

> **time** A dimension that enables two otherwise identical events that occur at the same point in space to be distinguished. The interval between two such events forms the basis of time measurement. For general purposes, the earth's rotation on its axis provides the units of the clock and the earth's orbit around the sun provides the units of the calendar. For scientific purposes, intervals of time are now defined in terms of the frequency of a specified electromagnetic radiation.
>
> In physics, since the publication of the special theory of relativity in 1905, Einstein has frequently been said to have abandoned the concept of absolute time. In this context absolute time is taken to mean "time that flows equably and independently of the state of motion of the observer". Time dilation effects and the collapse of absolute simultaneity mean that absolute time in this sense cannot be applied to the measurement of an interval of time.
>
> Although philosophers tend to describe Einstein's work on relativity as the beginning of a 20th century revolution in science, many of these 'revolutionary' concepts were not entirely original. In 1898, for example, Jules Poincaré[8] (1854-1912), the French mathematician, questioned the concept of absolute simultaneity, commenting that "we have no direct intuition about the equality of two time intervals." Poincaré was also aware of the need to consider local time for a given observer. In 1904, he observed that clocks synchronized by light signals sent between observers in uniform relative motion "will not mark the true time", but, rather, "what one might call the local time".
>
> A frequent misconception is that the theory of relativity removes absolute time from mechanics. This is true for the measurement of time as discussed above, but not for time itself. Newton's definition of absolute time is essentially a philosophical concept. Indeed, challenges to this concept in Newton's lifetime were usually made on philosophical, rather than experimental, grounds. Newton never claimed that one could measure absolute time; this absolute quantity had to be distinguished from the "sensible measures" used in "ordinary affairs."
>
> In Einstein's view of the universe, descriptions of a physical phenomenon need to be fully relativistic, requiring Lorentz transformations between the coordinates of systems in uniform relative motion. Contrary to popular belief, Newtonian mechanics was not based on absolute space and time and was fully relativistic, but in the Galilean sense; that is, Galilean transformations were required between the coordinates of systems in uniform relative motion.
>
> In considering simultaneity Einstein made use of a thought experiment.[9] As a result of this experiment in Einstein's view, the concept of absolute simultaneity has to be abandoned. His universe is causal, and in a causal universe, there is no such thing as simultaneity as there are no simultaneous events.[10] Events have a definite order based on their causal sequence, which cannot be changed. This is what Newton meant by absolute time. Without making a direct statement, Einstein effectively introduced a third postulate in his theory of relativity; that no information can be transmitted faster than the speed of light. For both Newton and Einstein absolute time is really the absolute order of events, determined by causality, and not the measurement of time, which is the subject of ordinary observation.

We will take a look and see if the numerous 'frequent misconceptions' noted in the above are indeed misconceptions. Most particularly, we will have to take a look at this idea of 'absolute time' and see whether or not Newtonian mechanics was 'based' on it. If one idea is key to the

[8] Poincaré's full name is Jules Henri Poincaré. He commonly went by 'Henri Poincaré.'

[9] 'Thought experiment' was what Einstein called it. What Einstein actually did was ask what would be the rational consequence of a situation allowed under Newtonian mechanics, and what he found was a contradiction between "Galilean relativity" (as the *Dictionary* calls it) and the theory of electromagnetism.

[10] This is carelessly worded. 'Absolute spontaneity' means that two events that are 'spontaneous' in the observation of one observer are also spontaneous in the observations of every other observer. Einstein did indeed rule this out. But 'local spontaneity' ('relative' spontaneity) *is* permitted in Einstein's theory.

context and explanation of another idea, it will not do to dismiss the former merely because it is 'philosophical,' even if this idea is regarded by people living in latter days to be embarrassing. It is true enough that Newton's absolute time is not needed in carrying out the practice of physics in solving 'mechanics problems,' and in this sense Newtonian mechanics is not 'based' on absolute time. But is the justification for the ontological validity of the objects of Newton's mechanics based at least in part on 'absolute' time? This we shall have to see for ourselves.

As for 'Einstein's universe' being a 'causal' universe, we might well ask, "What's wrong with that?" (since the quote above at least seems to have the flavor of being censorious). I'm guessing here, but I think the point being made is that this 'causal universe' is at odds with what many view as the 'non-causal nature' of the quantum theory, and Einstein did indeed turn his back on the probabilistic theory championed by the 'Copenhagen School' under Bohr. We also can ask what is meant by saying that Einstein "effectively introduced a third postulate" without "making a direct statement" of it. It is true that physics takes the view that 'information' is 'passed' between 'particles' (although physics does not commit itself to a technical definition of what is meant by 'information'; presumably 'information' in this context just means 'effects'). This is the role of the boson particles in modern physics. It is also true that Einstein's theory says that no 'particle' can ever be accelerated from a state of rest (relative to some observer) to a velocity exceeding the speed of light (relative to any observer). But it is also true that *if* there existed a 'particle' having a velocity greater than the speed of light (relative to *every* observer), that 'particle' could never be de-accelerated to a velocity below the speed of light (relative to *any* observer). Strictly speaking, then, the relativity theory does *not* 'rule out' the 'transmission of information' between 'particles' faster than the speed of light because the theory is silent on the problematical 'existence' of such 'particles.'[11] Dictionaries should not play at revisionist history.

What has this to do with the question of time? Issues of 'cause and effect' always involve, in one way or another, issues of 'time' because 'causation and effect' always implies some sort of succession of events. Officially at least, most physicists avoid talking about 'causality' any more, calling 'causality' a 'philosophical concept' (and here we see the lingering aftermath of positivism). If one says 'information is transmitted between particles' instead of 'effects are

[11] Such a hypothetical 'particle' is called a 'tachyon' in the language of physics. Tachyon particles have never been experimentally observed, and so far as I know Einstein never speculated on them, although it seems from the 1905 paper that he did not consider it possible that anything could travel faster than light and that such an idea was physically meaningless. Quantum electrodynamics does allow 'photons' to have any relative velocity, including one greater than the established value for the velocity of light, but QED theory does not say that such 'faster than light light-particles' are *accelerated* from below to above the 'speed of light.' The 'information issue' becomes pertinent due to experiments that have been conducted to refute what is known as the 'local hidden variables' postulate (see F. Rohrlich, "Facing quantum mechanical reality," *Science*, vol. 221, no. 4617, Sept. 23, 1983, pp. 1251-1255).

communicated from one particle to another' one can avoid being called on to speak to 'the causality problem' by means of a simple sophistry, because if we do not say 'effect' we do not have to say 'cause.'

But let us get down to cases. We begin with Newton. In the first scholium of the Definitions section of his *Principia* Newton writes,

> Hitherto I have laid down the definitions of such words as are less known and explained the sense in which I would have them to be understood in the following discourse. I do not define time, space, place, and motion, as being well known to all. Only I must observe, that the common people conceive those quantities under no other notions but from the relation they bear to sensible objects. And thence arise certain prejudices, for the removing of which it will be convenient to distinguish them into absolute and relative, true and apparent, mathematical and common.
>
> I. Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year.
>
> II. Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces; which our senses determine by its position to bodies; and which is commonly taken for immovable space, determined by its position in respect of the earth. Absolute and relative space are the same in figure and magnitude; but they do not always remain numerically the same [NEWT1: 8].

Newton continues with similar explanations for absolute and relative place and motion. 'Philosophical concepts' they may be, but Newton felt it important to spell out the differences between his 'absolute quantities' and their 'common' counterparts which were the sources of 'certain prejudices.' Could we simply tear out these pages of *Principia* and continue onward without losing anything crucial? Or should we suppose that Newton had a very good reason for spelling out these differences even though time, space, etc. are "well known to all"? Our first clue comes a few paragraphs later.

> Absolute time, in astronomy, is distinguished from relative, by the equation or correction of the apparent time. For the natural days are truly unequal, though they are commonly considered as equal, and used for a measure of time; astronomers correct this inequality that they may measure the celestial motions by a more accurate time. It may be that there is no such thing as an equable motion, whereby time may be accurately measured. All motions may be accelerated and retarded, but the flowing of absolute time is not liable to any change. The duration of perseverance of the existence of things remains the same, whether the motions are swift or slow, or none at all: and therefore this duration ought to be distinguished from what are only sensible measures thereof; and from which we deduce it, by means of the astronomical equation [NEWT1: 9]

The absolute time Newton sets down here is nothing else than an ontological pronouncement. He presents it with no attempted proofs; it is an ontological acroam *which is to be the basis by which other truths are to be ascertained*. Newton's absolute space is similarly such an acroam.

> As the order of the parts of time is immutable, so also is the order of the parts of space. Suppose those parts to be moved out of their places and they will be moved (if the expression may be allowed) out of themselves. For times and spaces are, as it were, the places as well of themselves as of all other things. All things are placed in time as to order of succession; and in space as to order of situation. It is from their essence or nature that they are places; and that the primary places of things should be movable is absurd. These are therefore the absolute places; and translations out of those places are the only absolute motions.
>
> But because the parts of space cannot be seen, or distinguished from one another by our senses, therefore in their stead we use sensible measures of them. For from the positions and distances of things from any body considered as immovable we define all places; and then with respect to such places, we estimate all motions, considering bodies as transferred from some of these places into others. And so, instead of absolute places and motions, we use relative ones; and that without any inconvenience in common affairs; but in philosophical disquisitions, we ought to abstract from our senses and consider things themselves, distinct from what are only sensible measures of them. For it may be that there is no body really at rest, to which the places and motions of others may be inferred [NEWT1: 9-10].

In Feynman's quote at the beginning of this chapter we saw a frank admission that to a certain degree our meanings of such fundamental 'things' as 'time' are the products of arbitrary definitions. (Arbitrary does not mean 'capricious'; a lot of worry goes into establishing important and fundamental *operational* definitions in science; things that serve as *standards*, e.g. a certain platinum-iridium bar kept in Paris called the 'standard meter', are very carefully preserved for the use of science). As Margenau pointed out, ultimately these definitions are anchors which set the fundamental postulates of the science.

But Newton is dealing with an even tougher issue, and to see this all we need do is remember the title of his book: *Mathematical Principles of Natural Philosophy*. The revolution Newton started in science succeeded beyond all measures of all his predecessors for one very important reason: Newton brought the full power of *mathematics* to bear in the solution of scientific problems, and he did so by means of something no one else had, so far as we know, ever seen before. It is called *the calculus*.[12] The fundamental postulates of physics are mathematical postulates. We have commented, and seen commented upon, several times in this treatise that the laws of physics are mathematical laws. But mathematics comes out of the minds of mathematicians and in its rationalist purity has nothing whatever that it owes to empirical nature. *Why should we think mathematics has anything to do with nature?* Newton is going to turn physics into *mathematical physics*. But how can he possibly do this? Why should we not, along with today's mathematical formalists, say "mathematics isn't *about* anything; it's just a game with rules." Newton's first task was *to justify the use of mathematics in the deducing of true and sure laws of nature*. And this is, indeed, a 'philosophical concept' of the most practical sort.

[12] In recent years there have been rumors and speculations that Archimedes of Syracuse might have invented the calculus. So far as I have heard, this has not yet been confirmed. But if he did, it died with him.

Today most scientists take the calculus for granted. Science and engineering students study it in their freshman year, and many students now take it in high school. It is almost impossible to overstate the importance of calculus in science. A long history of demonstrated successes has secured it a trusted place in science, so much so that the thorny issue its use presented at the dawn of modern science has been largely forgotten. We are today, I am sorry to say, ignorant of the *justification* for using the calculus to describe nature because science students are no longer required to read Newton. But Newton was aware of the issue, even if we today are not. And what issue is this? It is: *how to link the observable phenomena of nature to the unobservable objects of pure mathematics*. Newton began to forge this link as follows:

> It is a property of motion, that the parts, which retain given positions to their wholes, do partake of the motions of those wholes . . . A property, near akin to the preceding, is this, that if a place is moved, whatever is placed therein moves along with it; and therefore a body, which is moved from a place in motion, partakes also of the motion of its place. Upon which account, all motions, from places in motion, are no other than parts of entire and absolute motions; and every entire motion is composed of the motion of the body out of its first place, and the motion of this place out of its place; and so on, until we come to some immovable place . . . Wherefore, entire and absolute motions can be no otherwise determined than by their immovable places; and for that reason I did before refer those absolute motions to immovable places, but relative ones to movable places. Now no other places are immovable but those that, from infinity to infinity, do all retain the same given position one to another; and upon this account must ever remain unmoved; and do thereby constitute immovable space [NEWT1: 10-11].

In this discussion of 'places' we can see the shadow of Aristotle. This is, of course, only the beginning of the argument. 'Apparent quantities' – relative time, relative space, relative motion, and so on – might or might not give us, by our observations, true facts regarding nature. But, Newton tells us, there is a logical and unbreakable linkage between these appearances and what we seek to understand. To use a metaphor, the path exists; it's up to us to somehow find it.

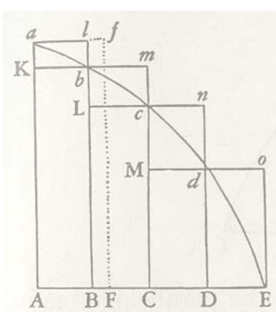
> Wherefore relative quantities are not the quantities themselves, whose names they bear, but those sensible measures of them (either accurate or inaccurate), which are commonly used instead of the measured quantities themselves. And if the meaning of words is to be determined by their use, then by the names time, space, place, and motion, their measures are properly to be understood; and the expression will be unusual and purely mathematical, if the measured quantities are themselves meant. On this account, those violate the accuracy of language, which ought to be kept precise, who interpret these words for the measured quantities. Nor do those less defile the purity of mathematical and philosophical truths, who confound real quantities with their relations and sensible measures [NEWT1: 12].

Note very carefully the distinction here between 'measures' (e.g. relative time) and 'measured quantities' (e.g. absolute time). Newton has just told us flat out: When we express physics mathematically, the symbols in the mathematical expressions mean the 'absolute, true, and mathematical' time, space, etc. The mathematics is very much indeed 'about something.'

It is obvious that this ontology is that of the transcendent realism that has always attended the copy-of-reality hypothesis in all systems of metaphysics prior to Kant. In Chapter 17 it was noted that Newton reified space in his theory, and he does likewise for time. But for the moment our concern centers on the role of his absolute quantities in the development of the outlook of physics. And lest we doubt that it is to link observables to 'true reality' that Newton introduces these absolute quantities, let us consider his own words:

> It is indeed a matter of great difficulty to discover, and effectually to distinguish, the true motions of particular bodies from the apparent; because the parts of immovable space, in which these motions are performed, do by no means come under the observations of our senses. Yet the thing is not altogether desperate, for we have some arguments to guide us, partly from the apparent motions, which are the differences of the true motions, partly from the forces, which are the causes and effects of the true motions . . . But how we are to obtain the true motions from their causes, effects, and apparent differences, and the converse, shall be explained more at large in the following treatise. For to this end it was that I composed it [NEWT1: 12-13].

The 'true motions' are those which are expressed mathematically, and Newton's calculus is the means for obtaining knowledge of these 'true motions.' Book One of the *Principia* begins with the introduction of the calculus, and Newton's argument, while symbolically modified in today's notation, is in all essentials the same argument presented to students today. He called his method 'the method of first and last ratios of quantities.' The arguments are geometrical, which is the usual method used by mathematicians of Newton's day. The figure shown on the left is an example taken from the *Principia* as an illustration of the formal argument. Newton began by introducing several lemmas (simple theorems). The first was: *Quantities, and the ratios of quantities, which in any finite time converge continually to equality, and before the end of that time approach nearer to each other than by any given difference, become ultimately equal.*



In today's mathematics, since the dominance of formalism, words like 'quantities' and 'time' as used here would be replaced by abstract, contentless ideas having no 'material' import but only a purely formal definition. Newton, however, means us to take these terms seriously; the quantities are spaces, lengths, areas, motions – and include both 'absolute' and 'common' quantities. When he says 'time' he does not mean an abstract parametric variable but *time itself*. Mathematics, then as now, is a language, but in the 17th century it was not a formal and contentless computer language.

Beginning with diagrams such as the one shown above, Newton proceeds to reduce the bases of the parallelograms and add more of them in a manner familiar nowadays to any college freshman taking calculus. The argument has changed little, except 'philosophically,' since Newton first set it down in the *Principia*. In his second lemma he tells us, *then if the breadth of* *those parallelograms be supposed to be diminished, and their number to be augmented* in infinitum*, I say that the ultimate ratios which the inscribed figure* AKbLcMdD, *the circumscribed figure* AalbmcndoE, *and curvilinear figure* AabcdE, *will have to one another are ratios of equality* [refer to the figure above for this lemma]. This lemma details the procedure which, in principle, finds the area under the curve AabcdE by the operation known as integration.

Now, the process of 'diminishing and augmenting' the parallelograms eventually takes us to a point where these parallelograms are no longer observable. (In the language of today, their base lengths Δx approach zero). Newton calls these 'evanescent parallelograms.' (He will later do this same operation on other geometrical figures, and these, too, will be labeled 'evanescent'). Four corollaries follow from his first three lemmas:

> Cor. I. Hence the ultimate sum of those evanescent parallelograms will in all parts coincide with the curvilinear figure.
> Cor. II. Much more will the rectilinear figure comprehended under the chords of the evanescent arcs *ab, cd, &c.,* ultimately coincide with the curvilinear figure.
> Cor. III. And also the circumscribed rectilinear figure comprehended under the tangents of the same arcs.
> Cor. IV. And therefore these ultimate figures (as to their perimeters *ac*E) are not rectilinear, but curvilinear limits of rectilinear figures [NEWT1: 26].

Why are these lemmas, corollaries, and the others which follow of *physical* import? In the first place, there is a practical importance (assuming one thinks it is important to describe such things as the areas under curves, etc.).

> These things which have been demonstrated of curved lines, and the surfaces which they comprehend, may be easily applied to the curved surfaces and contents of solids. These Lemmas are premised to avoid the tediousness of deducing involved demonstrations *ad absurdum*, according to the method of the ancient geometers. For the demonstrations are shorter by the method of indivisibles; but because the hypothesis of indivisibles seems somewhat harsh, and therefore the method is reckoned less geometrical, I chose rather to reduce the demonstrations of the following Propositions to the first and last sums and ratios of nascent and evanescent quantities, that is, to the limits of those sums and ratios, and so to premise, as short as I could, the demonstrations of those limits. For hereby the same thing is performed as by the method of indivisibles; and now those principles being demonstrated, we may use them with greater safety. Therefore if hereafter I should happen to consider quantities as made up of particles, or should use little curved lines for right ones, I would not be understood to mean indivisibles, but evanescent divisible quantities; not the sums and ratios of determinate parts, but always the limits of sums and ratios; and that the force of such demonstrations always depends on the method laid down in the foregoing Lemmas [NEWT1: 31].

But there is another, and profoundly more important, consequence of Newton's method. By starting with inscribing and circumscribing 'nascent quantities' and following this limiting process down to 'evanescent quantities,' *the mathematical description becomes that of the absolute quantities*. Newton has, he believes, captured the unobservable through an operation performed upon the observable 'common' quantities.

> Perhaps it may be objected, that there is no ultimate proportion of evanescent quantities; because the proportion, before the quantities have vanished, is not the ultimate, and when they are vanished, is none. But by the same argument it may be alleged that a body arriving at a certain place, and there stopping, has no ultimate velocity; because the velocity, before the body comes to the place, is not its ultimate velocity; and when it has arrived, there is none. But the answer is easy; for by the ultimate velocity is meant that with which the body is moved, neither before it arrives at its last place and the motion ceases, nor after, but at the very instant it arrives; that is, that velocity with which the body arrives at its last place, and with which the motion ceases. And in like manner, by the ultimate ratio of evanescent quantities is to be understood the ratio of the quantities not before they vanish, nor afterwards, but with which they vanish. In like manner the first ratio of nascent quantities is that with which they begin to be. And the first or last sum is that with which they begin and cease to be (or to be augmented or diminished). There is a limit which the velocity at the end of the motion may attain, but not exceed. This is the ultimate velocity. And there is the like limit in all quantities and proportions that begin and cease to be. And since such limits are certain and definite, to determine the same is a problem strictly geometrical. But whatever is geometrical we may use in determining and demonstrating any other thing that is also geometrical.
>
> It may also be objected, that if the ultimate ratios of evanescent quantities are given, their ultimate magnitudes will be also given; and so all quantities will consist of indivisibles, which is contrary to what Euclid has demonstrated concerning incommensurables in the tenth book of his *Elements*. But this objection is founded on a false supposition. For those ultimate ratios with which quantities vanish are not truly ratios of ultimate quantities, but limits towards which the ratios of quantities decreasing without limit do always converge; and to which they approach nearer than by any given difference, but never go beyond, nor in effect attain to, till the quantities are diminished *in infinitum*. This thing will appear more evident in quantities infinitely great. If two quantities, whose difference is given, be augmented *in infinitum*, the ultimate ratio of these quantities will be given, namely the ratio of equality; but it does not from thence follow, that the ultimate or greatest quantities themselves, whose ratio this is, will be given. Therefore if in what follows, for the sake of being more easily understood, I should happen to mention quantities as least, or evanescent, or ultimate, you are not to suppose that quantities of any determinate magnitude are meant, but such as are conceived to be always diminished without end [NEWT1: 31-32].

Newton was right in anticipating both that 'certain prejudices should arise' due to people's 'notions of common quantities' and that there would be 'objections' to his method of first and last ratios. Criticisms of his method arose almost at first publication.

The flavor of these criticisms is perhaps most easily understood if we employ a more modern notation. Suppose the function $f(t)$ represents some 'true quantity' in which we are interested. We will go ahead and interpret t as denoting absolute time. We will let δ denote some difference between absolute time and 'common' time. By Newton's method of ratios, we inscribe and circumscribe $f(t)$ and take the ratio of the difference to δ, i.e.

$$\dot{f}(t) \equiv \frac{f(t+\delta/2) - f(t-\delta/2)}{\delta} \ .$$

We now allow δ to be diminished *in infinitum*, i.e. taken in the limit to zero. Newton's critics objected that this is nothing else than 0/0, an undefined operation in mathematics. Let us suppose that $f(t) = t^3$. Then

$$\dot{f}(t) = \frac{3t^2\delta + \delta^3/4}{\delta} \ .$$

Now clearly the numerator is in fact zero when δ is zero, as is the denominator. But so long as δ is not zero we have $\dot{f}(t) = 3t^2 + \delta^2/4$, and this expression goes to $3t^2$ in the limit as $\delta \to 0$ (which is the correct calculus result). What Newton claims is 'dividing out' the δ in the denominator is a legal mathematical operation that does not change the result at the point where δ reaches zero. His critics claimed that dividing out the δ term was *not* legitimate for $\delta = 0$ because that division *is* none other than 0/0.

According to mathematics as it was in Newton's day, his critics were on very solid ground. Newton in point of fact had no *mathematical* defense to 'legitimize' his operation. But Newton was not really arguing his case *from mathematical grounds* in justification of the 'final step' in his limit argument. He was arguing his case on the *ontological* ground that 0/0 is 'undefined' in mathematics but that *in physics there is a definite limit to be had*, namely that the ratio would converge to the 'absolute quantity' in question. (If $f(t)$ represents the position of a body as a function of time, the ratio above gives its velocity as a function of time). This is the real importance of Newton's absolute time, absolute space, etc. in his theory. ***It justified the calculus***.

Newton's leading critic after the publication of the *Principia* was none other than George Berkeley. Berkeley's *The Analyst*, published in 1734 some seven years after Newton's death, contained a devastating attack on 'infinitesimals.' But, nonetheless, Newton's theory, even with the 'mathematically defective infinitesimal calculus,' was enjoying a stunning success, exceeding by far in scope and fecundity any previous physical theory. In the nineteenth century, after positivism succeeded in driving 'metaphysics' out of the practice of science, the practical success of Newton's physics, and the calculus upon which it was based, was simply too great to ignore, but the ontological argument, being 'philosophical,' could not be tolerated. This almost surely was a major factor in the mathematical developments of the time, particularly the work of Weierstrass and his 'epsilon-delta' method, which seemed to drive 'infinitesimals' out of mathematics once and for all[13] while still being able to retain the calculus on more 'acceptable' grounds. Weierstrass' method takes Newton at his word and formalizes the operation of 'limits.'

Given the overwhelming success of Newton's theory, and the calculus, it is understandable if the ontological underpinnings seem to be confirmed by it. However, this is not the whole story.

[13] "Infinitesimals" are today back once again in mathematics, and their legitimacy today is based on what is known as 'non-standard analysis.'

As we have already discussed, Newton's ontology eventually led to predictions that were contradicted by the Michelson-Morley experiment and which were resolved by Einstein's relativity theory. In his 1905 paper Einstein wrote,

> If we wish to describe the *motion* of a material point, we give the value of its coordinates as functions of the time. Now we must bear carefully in mind that a mathematical description of this kind has no physical meaning unless we are quite clear as to what we understand by 'time.' We have to take into account that all our judgments in which time plays a part are always judgments of *simultaneous events*. If, for instance, I say, "That train arrives here at 7 o'clock," I mean something like this: "The pointing of the small hand of my watch to 7 and the arrival of the train are simultaneous events."
>
> It might appear possible to overcome all the difficulties attending the definition of 'time' by substituting "the position of the small hand of my watch" for "time." And in fact such a definition is satisfactory when we are concerned with defining a time exclusively for the place where the watch is located; but it is no longer satisfactory when we have to connect in time series events occurring at different places, or – what comes down to the same thing – to evaluate the times of events occurring at places remote from the watch.
>
> We might, of course, content ourselves with time values determined by an observer stationed together with the watch at the origin of the coordinates, and coordinating the corresponding positions of the hands of the watch with light signals, given out by every event to be timed, and reaching him through empty space. But this coordination has the disadvantage that it is not independent of the observer with the watch or clock, as we know from experience. We arrive at a much more practical determination along the following line of thought.
>
> If at the point A of space there is a clock, an observer at A can determine the time values of events in the immediate proximity of A by finding the positions of the hands which are simultaneous with these events. If there is at the point B of space another clock in all respects resembling the one at A, it is possible for an observer at B to determine the time values of events in the immediate neighborhood of B. But it is not possible without further assumption to compare, in respect of time, an event at A with an event at B. We have so far only defined an 'A time' and a 'B time.' We have not defined a 'common time' for A and B, for the latter cannot be defined at all unless we establish *by definition* that the 'time' required by light to travel from A to B equals the 'time' it requires to travel from B to A . . . We assume that this definition of synchronism is free from contradictions and possible for any number of points; and that the following relations are universally valid; –
>
> 1. If the clock at B synchronizes with the clock at A, the clock at A synchronizes with the clock at B.
>
> 2. If the clock at A synchronizes with the clock at B and also with the clock at C, the clocks at B and C also synchronize with each other.
>
> Thus with the help of certain imaginary physical experiments we have settled what is to be understood by synchronous stationary clocks located at different places, and have evidently established a definition of 'simultaneous' or 'synchronous' and of 'time.' The 'time' of an event is that which is given simultaneously with the event by a stationary clock located at the place of the event, this clock being synchronous, and indeed synchronous for all time determinations, with a specified stationary clock.[1]

Now, this definition of Einstein's might be regarded as an 'absolute time' for the case where all these imaginary 'clocks' located throughout the universe are at rest ('stationary') *with respect to each other*. But it is certainly not the same thing as Newton's absolute time, which depended in no way on any observer or any other 'material thing.' And what happens if we have 'clocks' that

[1] A. Einstein, "On the electrodynamics of moving bodies," in *The Principle of Relativity*, (trs.) W. Perrett and G.B. Jeffery, NY: Dover Publications, 1952, pp. 33-65.

are in motion relative to one another? Einstein's theory shows that *no absolute time can be defined* for the case when the two clocks are moving relatively to each other with a uniform relative velocity (as determined by either of the observers whose 'clocks' we are discussing):

> Observers moving . . . would thus find that the two clocks were not synchronous, while observers in the stationary system would declare the clocks to be synchronous.
>
> So we see that we cannot attach any *absolute* signification to the concept of simultaneity, but that two events which, viewed from a system of coordinates, are simultaneous, can no longer be looked upon as simultaneous events when envisaged from a system which is in motion relatively to that system.[2]

There are indeed other consequences of the theory of relativity, especially when we consider Einstein's general theory, but they all come down to one thing: There is no way to define a single and unique 'absolute time' in the sense that Newton's ontology implicates. The *Oxford Dictionary* commentary we looked at earlier claimed that Einstein also had his 'absolute time,' but quite frankly I see no evidence to support this claim at all in Einstein's scientific works, nor do I think that what was claimed in that commentary about what Newton and Einstein meant by absolute time is supported by history. I am sorry to say I think the *Oxford* commentary is nothing more than a not-very-accurate revisionist history in this regard. In Newton's case, we have already seen that his ontological 'time' was not a mere adornment appended to his theory, but rather was a very fundamental factor in it.

In Einstein's case, it is true that he did not accept the 'non-causality' the view of the Copenhagen School of quantum theory promotes. But Einstein never mounted a counterattack based on any considerations involving 'absolute time.' What he did instead was try to pose counterexamples (contradictions) to show that the Copenhagen interpretation was wrong. Bohr responded to each of these attempts by refuting Einstein's counterexamples. In one very famous (among physicists) instance, Bohr showed that Einstein's example was wrong because Einstein himself had forgotten to take into account a result from his own relativity theory! So while it is true that Einstein's objections seem to have been 'philosophical,' I think all we can say is that the probabilistic interpretation of the quantum theory wouldn't fit in with whatever personal pseudo-metaphysical prejudices Einstein held. It would be more than a little interesting to know what Einstein would have thought of the quantum electrodynamics theory, with its 'particles moving backward in time' and its 'faster-than-light photons,' but unfortunately there is no way we will ever know.

But what about the calculus? If its validity depended upon Newton's absolute quantities, does the experimental contradiction of this ontology take down the calculus as well? This is quite

[2] *ibid.*

an interesting question. Mathematics contented itself that the problem of 'infinitesimals' was taken care of in the nineteenth century through Weierstrass' work. But what, exactly, does the 'epsilon-delta' method say in regard to this issue? We previously stated the mathematical form of 'continuity' in Chapter 16 (§6.2). What Weierstrass said was that our function $\dot{f}(t)$ 'exists' at *t* if there are some small positive numbers *c*, *δ* and *ε* for which $\left|f(t \pm c) - f(t)\right| < \varepsilon$ when $c < \delta$ for any choice of *ε*. In terms of our previous specific example, as *c* goes to zero we can always find such an *ε* and such a *δ*, and so Newton's ratio 'exists' (in the mathematical sense) but without an interpretation in terms of absolute physical quantities.

To the non-mathematician this seems to beg off from 'really' answering Berkeley's 0/0 objection, and I suppose in a way it does. As noted before, in Newton's day 0/0 was mathematically 'undefined.' What Weierstrass did was to define it, and to define it in such a way that it accords with Newton's calculus. It is through Weierstrass' 'new definition of continuity' that the calculus is 'saved' after the loss of Newton's 'absolute quantities.' A cynic might say that mathematics is 'cheating' in the sense that it simply 'defines the problem away.' There is some truth, too, in this charge. Mathematics, however, prefers to call this 'analysis' rather than 'cheating' and, under formalism, all this is okay because "mathematics isn't *about* anything except mathematics." The calculus has simply proven too valuable to throw away because of 'philosophical' objections. Interestingly enough, something of the flavor of Weierstrass' definition is contained in Newton's first Lemma: "Quantities, and the ratios of quantities, which in any finite time converge continually to equality, and before the end of that time approach nearer to each other than by any given difference, become ultimately equal." This can be regarded as a somewhat imprecise statement of Weierstrass' continuity definition, and it is an interesting speculation, but no more than a speculation, to say that Newton 'really' came up with Weierstrass' continuity idea two centuries before Weierstrass. But I think this gives Newton more credit than he has earned and would merely be revisionist history attempting to 'show' that Newton's 'natural philosophy' was not 'philosophical.'

§ 2. The Empirical Psychology of Time

With the exception of Bergson, the views on time we have just reviewed are products of rationalizing. With the exception of Augustine, for whom time was something subjective, and of Einstein, who's scientific stance on time was both positivist and pragmatic, time is regarded as external and reified as a thing. This objective time, as an object, is a *noumenon*, and at this late point in this treatise there is probably no need to repeat and stress that these ideas are transcendent speculations.

Even with Augustine, the subjectivity of time stems from an ontological view, namely the non-existence of the past (as a thing) and of the future. Bergson's 'duration' is somewhat trickier to pin down because sometimes he speaks of duration as something internal to the nature of living beings, yet at other times he seems to equate it, as 'the flux of pure duration,' to life or even 'existence in general.'

> I find, first of all, that I pass from state to state. I am warm or cold, I am merry or sad, I work or I do nothing . . . I change, then, without ceasing. But this is not saying enough. Change is far more radical than we are at first inclined to suppose.
>
> For I speak of each of my states as if it formed a block and were a separate whole. I say indeed that I change, but the changes seems to me to reside in the passage from one state to the next . . . Nevertheless, a slight effort of attention would reveal to me that there is no feeling, no idea, no volition which is not undergoing change every moment: if a mental state ceased to vary, its duration would cease to flow . . . My mental state, as it advances on the road of time, is continually swelling with the duration which it accumulates . . . Duration is the continuous progress of the past which gnaws into the future and which swells as it advances [BERG2: 1-4].

Our eminent empiricists, whose views we have reviewed above, are in agreement with one another that we do not *perceive* time through receptivity as an object of outer sense, although their views part on the issues of how we come to know about time and what we come to know about it. But if time is not an object of outer sense, it must then be a psychological object, and so we next examine the psychology of time.

§ 2.1 Time and William James

James begins his examination of time with introspection, and he comes to a conclusion that shares some points of similarity with the views of Augustine. But it also has some key differences.

> Let any one try, I will not say to arrest, but to notice or attend to, the *present* moment of time. One of the most baffling experiences occurs. Where is it, this present? It has melted in our grasp, fled ere we could touch it, gone in the instant of becoming . . . and it is only as entering into the living and moving organization of a much wider tract of time that the strict present is apprehended at all. It is, in fact, an altogether ideal abstraction, not only never realized in sense, but probably never even conceived of by those unaccustomed to philosophic meditation. Reflection leads us to the conclusion that it *must* exist, but that it *does* exist can never be a fact of our immediate experience. The only fact of our immediate experience is what Mr. E.R. Clay has well called "the *specious* present" [JAME2: 398].

For Augustine time existed entirely and only in the present. James tells us that we have no actual, direct experience of a crisp 'present' at all. We know events have happened; we know they are happening; we even know some things are going to happen but have not happened yet. But we do not know any 'somewhen' when happenings are immediately 'now.' The necessity we accord to the existence of 'the present' is a logical necessity; the present is that which joins the past to the future.

> In short, the practically cognized present is no knife-edge, but a saddle-back with a certain breadth of its own on which we sit perched, and from which we look in two directions into time. The unit of composition of our perception of time is a *duration*, with a bow and a stern, as it were – a rearward- and a forward-looking end. It is only as parts of this *duration-block* that the relation of *succession* of one end to the other is perceived. We do not first feel one end and then feel the other after it, and from the perception of succession infer an interval of time between, but we seem to feel the interval of time as a whole, with its two ends embedded in it. The experience is from the outset a synthetic datum, not a simple one; and to sensible perception its elements are inseparable, although attention looking back may easily decompose the experience, and distinguish its beginning from its end [JAME2: 399].

Although James might not appreciate my saying so, what he has just described is Kant's synthesis in continuity between moments in time (aesthetic Idea). We have previously said that the intuition at one moment in time 'grows out of' the previous moment. These moments, which are markings of reflective judgment and not parts *of* time, constitute the 'bow' and 'stern' of what James above called 'a duration.' The quintessence of the *modus* of succession is innovation, change in sensible representation in going from one moment in time to the next in the synthesis of apprehension. (The magnitude in the *Existenz* of this innovation is its *Critical* duration).

This innovation in the representation of sensibility is not yet cognition, which requires more than a minimum of intuition and, in comprehension, requires also the contribution of concepts. James goes on to say,

> And just as in certain experiences we may be conscious of an extensive space full of objects, without locating each of them distinctly therein; so, when many impressions follow in excessively rapid succession in time, although we may be distinctly aware that they occupy some duration, and are not simultaneous, we may be quite at a loss to tell which comes first and which last; or we may even invert their real order in our judgment. In complicated reaction-time experiments, where signals and motions and clicks of the apparatus come in exceedingly rapid order, one is at first much perplexed in deciding what the order is, yet of the fact of its occupancy of time we are never in doubt [JAME2: 399].

We will have to deal with what precisely it means to say that 'succession' is 'rapid' or 'slow.' We are, after all, attempting to build up a picture of what 'time' means under the Copernican hypothesis. In doing so, we can hardly use 'time' to define 'rapid' if 'rapid' is a predication we can make of succession (which is one of the three *modi* of subjective time). But with this *caveat* in mind, the experimental evidence to which James alludes does in fact show that test subjects in experiments such as the ones he goes on to describe do have difficulties, and even make errors, in 'keeping track' of the order of rapid perceptual events, particularly when these subjects do not know what to expect during the course of the experiment. Your author's optometrist seems to take some pleasure in demonstrating this to me each time I go in for an eye examination.

James went on to review some of the experimental data and psychological characteristics that emerged from studies that had been conducted in his day. He noted that there were marked differences in the characteristics of 'sense of duration' and those having to do with spatial perception. He also pointed out particular tendencies in perception, such as the tendency to break up monotonously-given series of sounds into rhythms of some sort. He cites a number of results obtained by other researchers, including Wundt, Dietze, and others.

> *Our sense of time*, like other senses, *seems subject to the law of contrast*. It appeared pretty plainly in Estel's observations that an interval sounded shorter if a longer one had preceded it, and longer when the opposite was the case.
>
> Like other senses, too, *our sense of time is sharpened by practice*. Mehner ascribes almost all the discrepancies between other observers and himself to this cause alone.
>
> *Tracts of time filled* (with clicks of sound) *seem longer than vacant ones* of the same duration when the latter does not exceed a second or two. This . . . becomes reversed when longer times are taken.
>
> There is a certain emotional *feeling* accompanying the intervals of time, as is well known in music. *The sense of haste goes with one measure of rapidity, that of delay with another*; and these two feelings harmonize with different mental moods [JAME2: 404-405].

What is meant by the 'sense of time' of which James here speaks? Is he speaking of some special sense like the sense of hearing or the sense of sight? Or is he speaking non-technically, such as when one speaks of a 'sense of desperation'? On the whole it seems he is using the latter sort of connotation for this 'sense of time' because he goes on to ask whether or not we have any 'sense' of raw, unfilled time (i.e. a 'sense of' or 'feeling for' time *per se*).

> Although subdividing time by beats of sensation aids our accurate knowledge of the amount of it that elapses, such subdivision does not seem at the first glance essential to our perception of its flow. Let one sit with closed eyes and, abstracting entirely from the outer world, attend exclusively to the passage of time . . . There seems under such circumstances as these no variety in the material content of our thought, and what we notice appears, if anything, to be the pure series of durations budding, as it were, and growing beneath our indrawn gaze. Is this really so or not? The question is important, for, if the experience be what it roughly seems, we have a sort of special sense for pure time – a sense to which empty duration is an adequate stimulus; while if it be an illusion, it must be that our perception of time's flight, in the experiences quoted, is due to the *filling* of the time, and to our *memory* of a content which it had a moment previous, and which we feel to agree or disagree with its content now.
>
> It takes but a small exertion of introspection to show that the latter alternative is the true one, and that *we can no more intuit a duration than we can intuit an extension devoid of all sensible content* . . . Our heart-beats, our breathing, the pulses of our attention, fragments of words or sentences that pass through our imagination, are what people this dim habitat. Now, all these processes are rhythmical, and are apprehended by us, when they occur, in their totality; the breathing and pulses of attention, as coherent successions, each with its rise and fall; the heartbeats similarly, only relatively more brief; the words not separately but in connected groups. In short, empty our minds as we may, some form of *changing process* remains for us to feel, and cannot be expelled. And along with the sense of the process and its rhythm goes the sense of the length of time it lasts. Awareness of *change* is thus the condition on which our perception of time's flow depends; but there exists no reason to suppose that empty time's own changes are sufficient for the awareness of the change to be aroused. The change must be of some concrete sort – an outward or inward sensible series, or a process of attention or volition [JAME2: 405-406].

In short, we never have a representation, either in empirical intuition or in affective perception, of pure, unfilled time *per se*. James is going to take this as evidence against Kant's idea of a 'pure intuition of time.' However, the pure intuition time is not something we 'sense' or 'feel' or 'perceive' as such; it is merely the form of inner sense that brings an organization to the matter of perception and not the perception itself. But we can forgive James mistaking Kant's meaning since it is an easy mistake to make and one that is frequently made.

We see in the quote above some degree of agreement with Aristotle's view of time. Change is "the condition on which our perception of time's flow depends," but, like Aristotle, James does not say 'the change *is* the time.' We should question exactly to what James refers when he speaks of "the sense of the length of time"; this seems to be an ill-definable concept in the context of 'senses' and 'feelings'. We will let Piaget clear this up for us a bit later. James next undertakes a description of the character of what we do perceive in the context of the time-like in perception.

> In the experience of watching empty time flow – "empty" to be taken hereafter in the relative sense just set forth – we tell it off in pulses. We say "now! now! now!" or we count "more! more! more!" as we feel it bud. This composition out of units of duration is called the law of time's *discrete flow*. The discreteness is, however, merely due to the fact that our successive acts of *recognition* or *apperception* of *what* it is are discrete. The sensation is as continuous as any sensation can be. All continuous sensations are *named* in beats. We notice that a certain finite "more" of them is passing or already past. To adopt Hodgson's image, the sensation is the measuring tape, the perception the dividing engine that stamps its length. As we listen to a steady sound, we *take it in* in discrete pulses of recognition, calling it successively "the same! the same! the same!" The case stands no otherwise with time [JAME2: 407].

This description is, of course, congruent with our theory of the marking of a moment in time.

James holds that "the feeling of past time is a present feeling." In this he and Augustine are in full agreement, although they come to this common view from quite different ontological grounds.

> Even though we *were* to conceive the outer successions as forces stamping their image on the brain, and the brain's succession as forces stamping their image on the mind, still, between the mind's own changes *being* successive and *knowing their own succession*, lies as broad a chasm as between the object and subject of any case of cognition in the world. *A succession of feelings, in and of itself, is not a feeling of succession. And since, to our successive feelings, a feeling of their own succession is added, that must be treated as an additional fact requiring its own special elucidation*, which this talk about outer time-relations stamping copies of themselves within, leaves all untouched.
>
> I have shown, at the outset of the article, that what is past, to be known as past, must be known *with* what is present, and *during* the "present" spot of time . . . There is thus a sort of *perspective projection* of past objects upon present consciousness, similar to that of wide landscapes upon a camera-screen.
>
> And since we saw a while ago that our maximum distinct *intuition* of duration hardly covers more than a dozen seconds . . . we must suppose that *this amount of duration is pictured fairly steadily in each passing instant of consciousness* by virtue of some fairly constant feature in the brain-process to which the consciousness is tied. *This feature of the brain-process, whatever it be, must be the*

> *cause of our perceiving the fact of time at all*. The duration thus steadily perceived is hardly more than the "specious present," as it was called a few pages back. Its *content* is in a constant flux . . . Meanwhile, the specious present, the intuited duration, stands permanent, like the rainbow on the waterfall, with its own quality unchanged by the events that stream through it [JAME2: 411-413].

We find in the quote just given the beginning of James' passage from the description of the features of our 'perception of the fact of time' to his attempt to correlate these features with characteristics of brain function. We will not belabor his speculations in this regard here, both because the knowledge of brain function was very limited in James' time, and because merely juxtaposing appearances of brain action with the perceptive features we tag with the object 'objective time' gets us no closer to the root of the matter.

We can also see that James is not attempting to put an 'ontological face' on 'time.' He makes no speculations that attempt to 'get behind' the psychological characteristics we link to the idea of 'time' in our conceptualization of experience. This is probably James' pragmatism showing itself, perhaps along with a bit of the positivism that held sway in science in his day. The center of his theory is the 'specious present' he calls 'duration,' and his thesis of tracking down characteristics in brain activity that can be likened to the psychological character of 'duration' is as close as he appears to be willing to come to an ontological pronouncement of the 'the fact of time' *per se*. From the viewpoint of pragmatism and of functionalism this is a proper scientific attitude to assume. If we are to build up a model of the nature of the pure intuition of time, the specification of such a model should be such as to ground the most rudimentary ideas we form of *objective* time. But we can see that James' work does not fully cover the scope of the development of this idea, and we will benefit from having behavioral facts to go along with the introspective features described above. For this, we turn to Piaget.

§ 2.2 Piaget and the Genesis of Objective Time

Piaget's developmental psychology of time appears primarily in two works: *The Construction of Reality in the Child* and *The Child's Conception of Time*. The former deals with the genesis of experience that lays the foundations for the later development of ideas of objective time. The latter deals with the stages of construction and the specific types of experiences that lead to and dress the child's idea of time. By this point in our treatise it will be no surprise to find that the evolution of the child's conception of time progresses in stages beginning with the entirely practical and progressing to the objectively theoretical. The infant and the toddler do not appear to 'think about time' as an object at all. In the beginning, any elementary concepts of time (that is, concepts that eventually become united in the scope of representations of an object we will call 'objective time') that might enter in to the child's activities serve no more than a coordinating practical role and are not set out and examined in their own right.

The child's representations and concepts that will eventually belong to objective time are obviously not accessible directly to the psychologist-observer. Equally clear is that, from Piaget's normative convention, it is going to prove to be very difficult to talk about 'time' without using our adult concepts of 'time.' This is the sort of circular character that inevitably attends the conceptualization of a time-object whose grounds must necessarily be laid to a pure form of intuition. But let us allow Piaget to tell us about this in his own words.

The Primitive Foundation

It probably takes no great deal of imagination on the reader's part to anticipate that partitioning out 'time' from observations of infantile behavior is a task attended by many difficulties, not the least of which is 'identification.' What part of observable behavior implicates a capacity for 'knowing time' at work in the mind of the infant (as opposed to some other capacity of intellect)? What approach must we take, and what precautions against any unwarranted transferal of our own adult concepts of time to the mind of the child must we employ?

> In a sense it can be said of time, as of space, that it is already given in every elementary perception; every perception lasts, just as every perception is extended[3]. But this first duration is just as removed from time properly so called as is the extension of sensation in organized space. Time, like space, is constructed little by little and involves the elaboration of a system of relations. These two constructions are correlative . . .
>
> It is this interconnection among the four fundamental categories of object, space, causality, and time which makes possible an analysis of time on the sensorimotor level of infantile intelligence. Without the relations of time with the other forms of organization of the universe it would be useless to try to reconstruct the temporal series which the child's mind elaborates, since consciousness of time is not externalized in the form of separable behavior as is consciousness of spatial relations. But if what we have thus far established with regard to objects, space, and causality has a temporal aspect, this may be disentangled by comparing the results obtained in each of these categories [PIAG2: 320-321].

This is the strategy for isolating and studying the 'temporal aspect' of infantile intelligence. Sensorimotor object, space, and causality are distinct constructions, but what they share in common is the bonding of a structuring 'in' time. (This 'makes sense' from the perspective of Kant's categories of Relation, wherewith 'object' implicates the *modus* of persistence in time, 'space' implicates the *modus* of coexistence in time, and 'causality' implicates the *modus* of succession in time). Said another way, time serves as the contact of object, space, and sensorimotor causality with consciousness.

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[3] What Piaget refers to here as perception is what we call intuition, not affective perception.

> In a general way, the formation of time is then parallel to that of space and complementary to that of objects and causality. In other words, it proceeds equally from the immediacy characteristic of radical egocentrism to a forming of relationships such that the mind is freed from its personal point of view and located in a coherent universe. At its point of departure time is intermingled with the impressions of psychological duration inherent in attitudes of expectation, effort, and satisfaction, in short, with the activity of the subject himself. This duration is subsequently put into closer and closer relations with events of the external world. At its point of arrival, time is promoted to the rank of an objective structure of the universe as such. The sequence of the subject's acts is thus inserted, as a lived sequence, in the series of remembered events constituting the history of the environment; this history does not remain incoherent, as before, with its fragments attached to current action conceived as the sole reality.
>
> Beginning with sensorimotor intelligence, time necessarily transcends pure duration and, if this duration is indeed at the source of time, it would never become truly temporal without a spatialization and an objectification inseparable from the entire intellectual activity [PIAG2: 321-322].

Piaget's last statement was likewise Kant's view [AK18: 306-307, 312-313, 612-613, 633]. We will shortly see what Piaget means by 'psychological duration.' We can for now get by with understanding this term in the manner James used earlier. The descriptive difficulties we are encountering are to be expected when one tries to describe a *primitive* factor such as 'time.' We are faced here with an issue not unlike the one we encountered in Chapter 3, when we noted that the only way to describe 'representation' was by making a representation. In that case, we saw that to represent representation we had to represent 'it' in terms of results (compositions and connections). Similarly, **a valid idea of time is *practical*, i.e., defined by what we say it *does***.

Piaget presents the construction of objective time through stages of representations. At each stage this description takes the form of particular types of 'series,' which he names the practical series, the subjective series, the objective series, and the representative series. As we examine these in turn, let us go into this examination with the provisional hypothesis that we are going to find the idea of a 'series' to be, in terms of general representation, the idea of 'connection' in the representation of time. This provisional hypothesis stems from the idea that the objective validity of the idea of 'time' is vested in a process of synthesis in sensibility, which, as the making of a representation of sensibility, requires a form of *nexus* for a matter of composition.

Now, for the first two stages of sensorimotor intelligence (reflexes and the first habits), Piaget tells us that the 'temporal' series is a practical series. This type of series we look at as the manifestation of a practical 'time itself' (to use Piaget's words).

> The only question that can be asked in connection with the reflex stage and the stage of primary circular reactions is whether these primitive behavior patterns fulfill the conditions which the remaining observations will show to be necessary for the arrangement of moments in time and for measuring duration. No direct analysis of the initial forms of time being possible, we must be satisfied to compare what the child at the first two stages does or does not do with what the child of the subsequent stages is capable of performing, from the temporal point of view.
>
> As early as his reflex activity and the formation of his first habits, the nursling shows himself capable of two operations which concern the elaboration of the temporal series. In the first place, he

> knows how to coordinate his movements in time and to perform certain acts before others in regular order. For instance, he knows how to open his mouth and seek contact before sucking, how to steer his hand to his mouth and even his mouth to his thumb before putting the thumb between his lips, etc. In the second place, from the third stage, he knows how to coordinate his perceptions in time and even how to utilize one perception as signal for another. Thus from 0;1 (22) to 0;2 (12) [PIAG1, observations 44-49] the child knows how to turn his head when he hears a sound and to try to see what he has heard . . . What do these behavior patterns imply from the point of view of consciousness of time? [PIAG2: 322].

Piaget warns us at this point that we must take precautions to avoid interpreting these behaviors in adult terms, i.e. in terms of how the adult thinks about his idea of time. To the psychologist-observer it is 'obvious' that there is a series of events at work here. But this does not mean that to the child there is any distinct grasp of a 'series' in objective representation or even consciousness that a 'series' is 'in action' here.

We have every reason to think that, at this stage, the sensorimotor scheme functions as a primitive 'unit of activity' so far as the child's objective perception is concerned. There is a practical orientation to the 'temporal series' at this stage that parallels the practical spatial 'group' of the first two stages.

> Concerning space, we have seen that . . . the nursling is confined to coordinating his own movements spatially without conceiving of them as spreading out into groups, externalized and related to things. The practical group of the first two stages is precisely this space in action. Thus it precedes all perception and all representation of groups which define space as a relation between objects and as a common, homogeneous, and external environment. From this point of view, everything leads us to believe that the initial groups of displacements which remain purely practical from the point of view of space remain so also from the point of view of time. In other words, the child can manage to regulate his acts in time without either perceiving or representing to himself any sequence or temporal series regulating the events themselves [PIAG2: 323].

Jumping ahead of ourselves a bit, at this stage all we can say is that the phenomenon of 'time' appears as nothing more or less than as an ability to *order innovations* in elementary schemes and produce, in the eyes of the observer, a *closure* in actions that defines the appearance of equilibrium. This is a hint that the objective validity of our idea of the pure intuition of time is going to be vested in the appearance of a *practical ability* to represent sensibility in terms of a practical order structure with closure in actions.

Piaget briefly reviews the characteristics that distinguish the first two stages of sensorimotor development and contrasts these with those of the third stage (the stage of secondary circular reactions). The feature that principally distinguishes the primary from the secondary circular reactions is that in the former the child's behaviors indicate that he perceives his actions as an undifferentiated whole, whereas in the secondary circular reaction there is a nascent differentiation between the perception of a condition and that of a consequence which marks the first beginnings of intentionality in behavior.

> We can call the circular reactions of the second stage "primary." Their character consists in simple organic movements centered on themselves (with or without intercoordination) and not destined to maintain a result produced in the external environment. So it is that the child grasps for the sake of grasping, sucking, or looking, but not yet in order to swing to and fro, to rub, or to reproduce sounds. Moreover the external objects upon which the subject acts are one with his action which is simple, the means being confused with the end. On the other hand, in the circular reactions which we shall call "secondary" . . . the movements are centered on a result produced in the external environment and the sole aim of the action is to maintain this result; furthermore it is more complex, the means beginning to be differentiated from the end, at least after the event [PIAG1: 157].

Within the primary circular reaction we can conclude that the child experiences a series of perceptions, but not that the child perceives this series to be a sequence.[4] This lack of recognition that the series of perceptions is a *sequence* is what is denoted in calling the series at this stage a practical series. The phenomenal behaviors in the first two stages bring us to a fundamental and primitive psychological fact:

> In a general way may we not assert that in the whole practical series there is effort, desire, hence expectation, feeling of dissatisfaction, then of satisfaction, in short, awareness of duration and of a sequence of states? This, we believe, is the crux of the matter. The preceding considerations do not in any way demonstrate that awareness of time is absent in the first two stages of intellectual evolution. All that we say is that there are not yet concepts of time applying to external phenomena nor is there a temporal field encompassing the development of events in themselves independently of personal action. But just as space begins as the simple practical coordination of body movements before it is constituted as a relationship between permanent objects and the body itself, so also time begins as simple duration immanent in the practical series before it is established as an instrument of ordination interconnecting external events with the subject's acts. Hence primitive time is not time perceived from without, but duration experienced in the course of the action itself.
>
> What is this duration? It is mingled with impressions of expectation and effort, with the very development of the act, experienced internally. As such it certainly fills the child's whole universe, since no distinction is yet given between an internal world and the external universe. But it does not comprise either a real 'before' or 'after,' which are always relative to events regulated in themselves, or a measure of intervals, which also depends on the formation of relationships between the actions and the guidemarks of the external world. It is therefore time itself, in its immediacy as well as its imprecision: simply the feeling of a development and of sequential directions immanent in the states of consciousness [PIAG2: 325-326].

This 'duration' Piaget describes here as 'time unperceived' and 'merely immanent' in the ordering of elementary perceptions. That we do in fact perceive representations in a sequence from one moment in time to the next – indeed, that we are even able to (eventually) conceive of a 'next' that follows a 'before' – is a primitive psychological fact, and the word 'time' (in the sense of Piaget's 'duration') therefore implicates nothing more nor less than this basic phenomenal fact. Piaget calls this primitive psychological fact 'duration'; Kant calls it (subjective) 'time'. As

[4] The observer, of course, cannot observe what manner of perception the child experiences. But the hypothesis that the child must experience a series of perceptions is consistent with every observable fact of his behavior at this stage. However, the hypothesis that the child perceives this series as a sequence, i.e. that he recognizes that 'one thing follows another,' appears to be inconsistent with second-stage behavior.

Object it is transcendental Object, necessary for the possibility of experience in the manner in which we know our experiences. The primitive psychological fact is the ground for the *Dasein* of this transcendental Object, and any ontology we build for it merely clarifies details of its *Existenz* and can not go beyond nor separate itself from this basic phenomenon. We cannot, with objective validity, seek for some *Ding an sich selbst* as its ground, no 'time-in-general' for which this 'duration' or 'subjective time' is a special case. On the contrary, **this 'duration' or 'subjective time'** ***is itself the ground and starting point for all other ideas of all other kinds of objective 'time.'*** Again, Critical epistemology takes precedence over ontology.

The Pre-Construction of Objective Time

From this point forward we will begin to see the ties of 'Piagetian duration' to the pure intuition of time. Piagetian duration is primitive. It implicates no real 'before' nor 'after' and ***its quintessence is to be an ordering function for perception***. It has no 'unit' and no cardinality. This function is immanent in all concepts of a series or a sequence, but it is not 'itself' a concept or an idea or an Idea. It is not even an Ideal. As a capacity of *nous* it belongs to the synthesis of sensibility and, logically considered, it is a 'practical verb': *to put in order*.

The first beginnings of the construction of *objective* time appear in the third stage of sensorimotor intelligence. It is marked by the development of the secondary circular reaction.

> The time characteristic of the first two stages is a practical time, interconnecting the sequential movements of the same scheme but unconscious in its unfolding and at most giving rise to the sensations of expectation, effort, arrival at a goal, etc., that are characteristic of purely psychological duration. Beginning with the third stage, on the contrary, this situation is modified to the extent that the child begins to act upon things and to make use of their interrelations through prehension of visual objectives. The temporal series thus transcend the purely practical relations subsisting between personal acts and gestures and are henceforth applied to external events. But this extension of time remains subordinate to one essential condition: it takes place only to the extent that these movements depend on personal action. In other words, time begins to be applied to the sequence of phenomena but in proportion as that sequence is due to the intervention of the child himself. It is this type of series we shall call subjective series [PIAG2: 327].

The third stage, Piaget tells us, marks the first observable appearance of actions that can be called 'intentional' and can, for the first time, be regarded as involving conceived objectives toward which actions appear to be directed. To use our terminology rather than Piaget's, the child now begins to actively make use of the ordering structure he builds into the manifold of concepts. Of course, the 'objectives' of the third stage are extremely limited. Piaget calls the third stage the stage of "procedures destined to make interesting sights last."

This newly exhibited ability to make use of concept structure (again using our terminology) is accompanied by other behaviors that mark the first evidence of the emerging of nascent object concepts, spatial relationships, and nascent concepts of causality. In the third stage,

> The object begins to be formed, but only to the extent that it emanates from the activity of the subject. So also causality, which at first is intermingled with the inner relationships of the act (those uniting desire with satisfaction), begins, from the third stage on, to be applied to things. But it is applied without being detached from personal activity. To the exact contrary, the causality of the third stage consists in a confused relationship of efficacy and phenomenalism such that personal action is conceived as sole cause, not only of the results which experience shows it is actually capable of producing, but also of any effect that emerges without objective contact with the subject. In these conditions the space of the third stage consists in a projection of practical groups into the perceptual field, but a field circumscribed by personal action alone . . . With regard to time, the following is true: the subjective series constitute an application of time to things, but to the extent that the sequence of events which occurs in the midst of things is governed by the subject. In other words, the child does not yet perceive the sequence of events which are independent of himself, that is, he is not yet capable of forming objective series. But he has transcended the level of merely experienced time; the subjective series thus form the transition between the practical series and the objective series.
>
> At the time secondary circular reaction begins it is not certain that it immediately necessitates an orderly arrangement of perceptions in time. The child is limited to seeing that this gesture produces that result and to reproducing the efficacious gesture as precisely as possible. The temporal arrangement required by such a behavior pattern begins, therefore, by being practical and does not at the outset presuppose a seriation of the perceptions themselves, in other words, an elaboration of subjective series . . . Once these elementary schemes of secondary circular reaction have been formed . . . there is no need, in order to make them work, to perceive the seriations which each of them comprises . . . The whole phenomenon is still conceived as an indissociable and almost immediate connection [PIAG2: 327-329].

Piaget's detailed observations, recorded in [PIAG1] and [PIAG2], illustrated pretty clearly the details of which he speaks here. But to provide an illustration of what he is talking about, imagine yourself getting into a car. This involves a complex and coordinated sequence of movements, but you do not concentrate on the details of executing these movements; you 'just do it' and, I assume, you accomplish this without hitting your head on the roof of the car, banging your knee on the steering wheel, or missing the car seat with your behind. Unless you are *deliberately* observing each movement in the sequence, it is 'all one motion' to you. There is "no need . . . to perceive the seriations." One difference here between yourself and the third-stage infant is that *you* are capable of visualizing this as an objective series and the child at this stage is not.

The new factor which seems to be at work in this stage is that, for the first time, the child seems to be capable of, however vaguely, marking out – or at least making use of – 'before' and 'after' in the construction of his schemes. This does not necessarily imply that the child has a *concept* of 'before' and 'after' and, in particular, does not imply that the child conceptualizes 'before' and 'after' prior to the action. The secondary circular reaction appears to exhibit the capacity for the infant to affect temporal ordering in sensibility (which our theory holds to imply the re-introduction of concepts into sensibility through the synthesis of reproductive imagination), but these appearances do not yet justify our concluding that an objective time has yet been conceptualized.

> Of course in attempting a hypothetical reconstruction of the inner reactions peculiar to these behavior patterns it is difficult to say with certainty when the purely practical series end and when the subjective series really begin. But if as heretofore we simply try either to describe things in terms of behavior patterns or to find in behavior the criterion of operations which may be conceived in terms of consciousness, we may believe that the subjective series are formed when secondary circular reaction bears upon two objects at once, and not solely on one. The presence of two separate objects, one of which conditions activity of the other, makes possible a perception of sequences in addition to a mere practical arrangement of sequential movements . . . It seems that the subject capable of the current behavior patterns utilizes only the concepts of before and after but is not yet capable of an orderly arrangement of the events themselves. It must be remembered that the temporal field is correlative to the elaboration of the causal series . . . From the point of view of time, this means that the child able to execute the first of these behavior patterns [the secondary circular reaction] is thereby capable of discerning a before and after in the results of his own acts, whereas only the second behavior pattern[5] will teach him to arrange events in order, that is, in so far as they are related to objects as such.
>
> This distinction may seem subtle. But by examining the mnemonic progress in this third stage we shall prove that it corresponds to real facts and is not merely an intellectual view [PIAG2: 330-331].

Piaget's remark that the subject "utilizes only the concepts of before and after" requires a comment. The observations do in no way prove that the child 'has' the *concept* of 'before' or 'after.' An entirely *practical use* of his manifold of concepts (by re-introducing them into sensibility) does not require a distinct concept of either 'before' or 'after' in order to use them. This is because the categories are notions of time-determinations insofar as the transcendental schemata are concerned. Ordering relations are therefore inherent in the manifold of concepts without any separate and distinct 'before' and 'after' concepts. Indeed, *practical use* provides the conscious *experience* we call 'before' and 'after', and the intuition of this experience must logically precede the formation of a *concept* of 'before' or 'after.' The series in the third stage is subjective, not objective.

> In short, at the third stage the child is able to perceive a sequence of events when he himself has engendered that sequence or when the before and after are related to his own activity, but if the perceived phenomena succeed each other independently of himself he disregards the order of occurrence. We do not thereby mean to maintain that he systematically upsets that order or is incapable of grasping some of its features. We merely claim that in such circumstances practical memory, connected with personal movements, takes precedence over every operation directed by external facts, and thus the objective structuring of time remains impossible [PIAG2: 334].

To sum up, the third stage of sensorimotor intelligence is, so far as objective time is concerned, the point in mental evolution where the 'raw materials' in experience necessary for the conceptualization of 'time' as an object are established. The infant no longer merely reacts to

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[5] By 'second behavior pattern' Piaget is referring to coordination of secondary schemes and their application to new situations. This is the behavior pattern that defines the fourth sensorimotor stage.

perceptions but has become capable in stage 3 of affecting the ordering in sensibility through the re-presentation of concepts in the synthesis of imagination. These concepts do not yet include a concept of objective time, but such a concept is not required for the merely practical arrangement of action sequences. It is enough that the *nexus* of the manifold of concepts is connected by judgments of Relation that are in their primitive essentials 'time-determinations' of Kant's transcendental schemata. The subjective and merely practical arrangement of action sequences in this stage provides the infant with new experience, based on the efficacy of his own actions, that provides new concepts which, eventually, serve as the starting point for the earliest ideas of objective time. We can say that the third stage is the pre-construction stage of objective time. In this stage the lumber and nails are delivered; the construction of the house follows.

The Beginning of Objective Time

It is in the fourth stage of sensorimotor intelligence that the child's behaviors reveal the presence of the earliest concepts that will become characteristic concepts of objective time. In stage 3 the child's behavior exhibits concepts that can be said to contain a kind of practical 'before' and 'after' distinction but, because the series is subjective rather than objective, we would have to say that distinct concepts of objective time are still lacking. Stage 4 marks the beginning of temporal concepts that are concepts of objects in their own right.

> Starting from the fourth stage the schemes acquired by means of the secondary circular reactions give rise to the kind of behavior patterns we have called application of familiar means to new situations. After having merely reproduced the movements leading to interesting results, the child becomes capable of inter-combining the schemes and of subordinating them to one another in the capacity of means and ends. Such progress brings about important results with regard to the development of time. A seriation of means and ends is possible only to the extent that the subject is able to arrange events in time; thus the subjective series peculiar to secondary circular reaction begin to be objectified.
>
> In short, the various behavior patterns characteristic of this stage converge and show how time, at first inherent in personal action alone, begins to be applied to events external to the self and thus to constitute objective series. But they converge and also show how limited this objectification still is. In the various realms to which we have just referred the objectification peculiar to the fourth stage remains relative and does not yet succeed in freeing itself from the primacy of personal activity. Thus the application of familiar means to new situations only constitutes a term of transition between simple circular reaction and the more complex behavior patterns that use the interrelationships of objects freely [PIAG2: 335-336].

The definitive behaviors, those that demonstrate temporal concepts of an objective character, are those which demonstrate that the child can recall past events and make use of this recollection in the application of his known schemes. Minimally at least these behaviors show that 'before' and 'after' are now consciously conceptualized as objects. On the other hand, it appears to be the case that the child has not combined these concepts in a single object 'time.' The behaviors are indicative of ordered episodes of events, and what appears to be objectified are events connected to more than personal activity. However, this freeing-up of the events from activities is tenuous, and resistance quickly brings back activities indicative of the subjective series. In this stage the child shows the ability to order events through reproductive imagination but not through productive imagination. (Events are 'recalled' but problematic 'new events' are not invented). From his experimental observations Piaget concludes,

> when the child searches in A for the object he has just seen disappear in B, the practical memory of the action linked with position A still prevails over all memory of the sequence of the displacements. In other words, the series again becomes subjective as soon as consideration of past actions exerted on the object reappears, whereas the series marking the beginning of this stage remain objective before intervention of the action. From the point of view of memory, this explains the paradox of residual reactions. The child sees the object disappear in A, looks for it and finds it, sees it disappear in B, explores position B for a moment and finally returns to A! What does this mean if not that the subject begins to elaborate an objective temporal series, this time involving two sequential displacements, but at the first practical obstacle the series again becomes subjective, that is, governed by the memory of actions which succeeded? . . . In short, the behavior patterns of the fourth stage relating to the object show that the child becomes capable of elaborating objective series and thus of arranging events in order of time, but that this acquisition remains unstable and subordinate to practical memory, in other words, to subjective series [PIAG2: 339].

There appear in the fourth stage 'temporal concepts' but these are not as yet combined in a single Object ('time *per se*'). In terms of our theory, these concepts are most likely obtained by abstraction from less distinct concepts of appearances, i.e. they are 'pulled out' of 'parent concepts' of the appearance of an event as concepts of coordination whose 'parent object' is the event and not an abstract time-object. In other words, these concepts 'understand the event' but they do not yet 'understand time.' Thus these concepts go into the making of concepts of physical cause-and-effect relationships but do not yet make a concept of objective time. Concepts of event ordering, in a word, are still *specific* and not yet *generalized*. Within the framework of transcendental logic, this evolution of the manifold of concepts lays necessary foundations for the later unification of these specific concepts under the Object 'time.'

> The main difference between the causality of the fourth stage and that of the third may be characterized as follows: when the child utilizes object A to act upon object B he no longer considers as cause of the movements of B the global movement he himself executes by utilizing A, but rather the activity of A as the center of relatively separate forces . . . This is simultaneously the beginning of the objectification of causality and of spatialization of causal connections. It goes without saying that such behavior patterns reveal from the point of view of time an ability to arrange events in objective series . . .
>
> But if time applies to things as such in proportion as causality is objectified and spatialized, the same reservation should be made apropos of such facts as apropos of observations relating to object and to space. The behavior patterns of the fourth stage mark only one phase of transition, and if causality begins to be externalized it still remains impregnated with the efficacy characteristic of personal activity. Thereafter, if the first objective series are observable during this period, the before

> and after they introduce into events do not give rise to systematic and connected arrangements. Time is not yet a common environment encompassing the totality of phenomena including personal action. It is only an extension into events of the subjective duration inherent in the activity of the child. In other words, the child's memory begins to enable him to reconstruct short sequences of events independent of the self, but it cannot yet retrace the entire chronology of the phenomena perceived in the external world nor, much less, permit an evaluation of the duration of intervals [PIAG2: 340-341].

The Embryo of Objective Time

In the fifth stage of sensorimotor development the concepts of the fourth stage, which served only for the formation of specific objective series, are brought together under a general concept that extends these concepts to take in the perceptual field. It is here that for the first time we can say there is evidence of objective time being represented. However, the condition for evoking this object-concept is still immediate perception and it appears to be the case that this object-concept does not divide, once and for all, objective time from objective causality as distinct objects. To put this in more Piagetian-like terms, 'time' at this stage is more of a mobile mental scheme *within* a Type II equilibration structure and is not yet a 'thing.' Metaphorically speaking, objective time at this stage is a mental embryo and its 'birth as a thing' is yet to come.

The fifth sensorimotor stage is the stage of the tertiary circular reaction and of "the discovery of new means through active experimentation." The entire character of this stage demonstrates that the child has become capable of creating and not merely re-creating an objective series. But this creation of the series is uncompromisingly 'empiricist' and not yet 'rationalist' insofar as evocations of the time-concept depend upon the instrument of immediate perception.

> With the advent of the behavior patterns of the fifth stage, most of which appear at about the age of one, time definitively transcends the duration inherent in personal activity, to be applied to things themselves and to form the continuous and systematic link which unites the events of the external world to one another. In other words, time ceases to be merely the necessary scheme of every action connecting subject with object and becomes the general environment encompassing the subject in the same capacity as the object. At the moment when objects cease to be mere displays at the subject's disposition and are organized into a substantial and permanent universe, when space is freed from the perspective peculiar to individual action and becomes established as the structure of that universe, and when causality transcends the efficacy of subjective activity and intercoordinates external phenomena, it is natural that time should obey an analogous evolutionary law and be constituted as objective reality, interconnected with physical causality, space, and permanence, and that it should incorporate the sequences emanating from personal action to which it had up to then been subordinate [PIAG2: 341].

The full flavor of time at this stage is best appreciated by studying Piaget's specific observations of stage-5 behaviors in [PIAG1] and [PIAG2]. I will not belabor the point here by repeating these details. Let it suffice to say that the child is now in possession of a concept structure instrumental for objective time, and he utilizes this concept structure as an instrument for thinking about appearances but not as an instrument of creative speculation. Put another way, the concept is used to refine the understanding of objective series but is still limited in its scope and is not yet applied to what Piaget calls the 'representative series.' Time at this stage is *in context* but does not yet *create context*.

The Birth of Objective Time

The sixth and last stage of sensorimotor intelligence is the stage of "the invention of new means through mental combination." The fifth stage was a 'stage of discovery' and in it the child actively performs 'experiments' to 'see what happens'; he is not yet able to imagine what *will* happen. It is not so much that he, to paraphrase Newton, "makes no hypothesis" but, rather, that the stage-5 child is not capable of 'making an hypothesis' beyond perhaps a vague 'if I do *this* something will happen." But 'what will happen' in a novel situation he cannot guess. To use a metaphor, the child is an 'experimental alchemist' and not an 'experimental physicist' at stage 5.

But at stage 6 the child's behaviors demonstrate a capacity for what some call 'evocative memory' – i.e. the use of past knowledge to reason through and predict what will happen without having to resort to actual experiments to get an idea of an expected outcome.

> The elaboration of the temporal series . . . is an attempt to go beyond the present for the sake of the immediate past and future. Consequently it is one attempt among others to free the mind from direct perception in favor of an intellectual activity capable of placing the data of that perception in a stable and coherent universe. But still more than the construction of objects, space, and causality, the elaboration of the temporal field requires the development of images . . . If we concede the results of our foregoing analysis concerning the functioning of intelligence . . . or the development of real categories, representation as evocation by image or by a system of signs of absent objects scarcely appears except in a sixth and final stage, which is contemporaneous with the progress of language . . . As soon as mental assimilation has been liberated from direct perception and is capable of functioning without external support, the objective series formed by the totality of the intellectual work which made that liberation possible are themselves extended into the future and the past in the form of representative series. The representative series are therefore only objective series extended by the intellectual operations peculiar to this sixth stage, and these operations, to the extent that they engender representations relating to time, are nothing other than evocative memory [PIAG2: 345-346].

Rather than being only one factor in the overall constitution of context, objective time in the sixth stage becomes a tool for putting together a new context, within which other concepts are placed in an imaginative series. Thus 'for the first time' concepts of objects can be subordinated in the sphere of a concept of a maxim for the synthesis of *comprehension*. This we may call a 'temporal field.' Piaget summarizes this genesis of objective time in sensorimotor intelligence, and this summary is worth quoting at length:

> The development of time, parallel to that of space, of object, and of causality, proceeds from an initial practical egocentrism such that events are set in order by a personal action immobilized in a continuous present, to an objectification such that events are linked together in an order which ends by encompassing personal duration and memories as episodes in this real history. Thus, during the first two stages, everything takes place as though time were completely reduced to impressions of expectation, desire, success, or failure. There is here the beginning of sequence linked with the development of different phases of the same act. But each sequence forms a whole isolated from the others, and nothing yet enables the subject to reconstruct his own history and to consider his acts as succeeding one another. Furthermore, each sequence consists in a gliding from the preliminary phase of desire or effort into the terminal phase of success or failure, experienced as a present without a past. Finally and most important, this completely psychological duration is not accompanied by a seriation of events as external and independent of the self . . . During the third stage . . . external events begin to be set in order as a function of secondary circular reactions, that is, of the beginnings of action upon things. But as the child still perceives the order of phenomena only when he himself has been the cause, he remains incapable of conceiving the chronology of his universe independently of his own action . . . therefore, to the child, objective time still does not exist. With the advent of the fourth stage this objectification progresses to the degree that the adjustment of means to ends in the intelligent behavior patterns entails a permanence of objects, an organization of groups of displacement, and a spatialization of causality which force the child to begin to arrange events in order, and is no longer only his personal actions. With the coming of the fifth stage this orderly arrangement of time no longer applies only to some privileged events but in principle to the whole perceptual field, without yet being extended to a more distant memory of the past, that is, to the evocation of moments of a time which elapsed without leaving a perceptible trace in the present. Finally, with the sixth stage, the objectification of the temporal series extends to representation, that is, the child, becoming capable of evoking memories not linked to direct perception, succeeds by that very fact in locating them in a time which includes the whole chronology of his universe. This does not in the least imply that this chronology is as yet well seriated or that the evaluation of duration is correct, but apart from the interpersonal relations these operations become possible because hereafter personal duration is placed in relation to that of things, and this makes possible both the orderly arrangement of moments in time and their measurement in relation to external points of reference [PIAG2: 347-349].

§ 2.3 Piaget and the Construction of Objective Time

The concept of objective time is seeded in the sensorimotor phase of intelligence, and this capacity for temporal ordering has and performs a function in the construction of experience. Temporal ordering is a ‘tool’ for empirical thinking and reasoning. The sensorimotor phase child ‘thinks with time’ but does not ‘think about time’ as a thing. But once time has been made an object, the idea of time, like concepts of other objects, will eventually be elaborated upon and perfected in the manifold of concepts.

Piaget’s documentation of this process is found in *The Child’s Conception of Time* [PIAG6]. Strictly speaking, we do not require the details of this elaboration for our present work. We already have what we need for the transcendental aesthetic of time. But it is still instructional to review some of the main points from Piaget’s theory, and this we will briefly do in this section.

After conceiving and using temporal maxims as ‘tools’ in the sensorimotor phase, the child continues to make use of this ‘tool’ in developing his understanding of the world around him as he views it with his attitude of uncritical realism. But here he finds his early concept of time often is not entirely up to the task at hand. This forces him to accommodate his time concept and make refinements to it, extending its sphere of concepts and its scope. For convenience sake, Piaget divides this sphere into 'physical' time and 'psychological' time. 'Physical' time is objective time as it is applied to external phenomena. 'Psychological' time is an objective time

> by which we mean the internal and representative co-ordination of the subject's actions, past, present, and future [PIAG6: 218].

He makes further more detailed subdivisions of 'time' for more specialized contexts, each of which takes on another descriptive adjective to set off the particular context within the general.

One of the theories he finds against is Bergson's reification of 'pure duration.' While Bergson agreed with what he thought was Kant's theory of space, i.e. that it was a 'mere intuition,' he disagreed with allowing the same interpretation for time-as-pure-duration. Bergson's 'time' ('pure duration') is ontologically a very strange thing but a thing nonetheless. Piaget calls this Bergsonian thing "a complete myth" [PIAG6: 218]. To use the terminology of this treatise, the *ens originarium* of time is the transcendental Subject and no ontology of time that denies this transcendental source of the time-object can have objective validity. Reified time is not a transcendental but rather a transcendent *noumenon*; it goes past the horizon of any possible experience. All understanding of the phenomenon of time must begin with the recognition that our knowledge of time is a practical by-product of equilibration and a feature of human experience. ***Objective time in Nature must therefore always be a 'mathematical time'*** and the questions that attend its valid conceptualization are those which ask: what is objectively valid in making a mathematical theory of time?

Getting back to Piaget and his findings: The fundamental phenomenon that goes into the conceptualization of objective time (and therefore the transcendental basis for objective validity in concepts of time) is the perception or imagination of motion.

> From all the preceding discussions, we have learned that time is the co-ordination of motions at different velocities – motions of external objects in the case of physical time, and of the subject in the case of psychological time. When we say motions, we are thinking of real motions, and not of the displacements or ideal movements of geometry. The latter are simply changes of position or 'placements,' in which the velocity can be neglected; that is why displacement is a spatial concept and why time only appears with real motions, i.e. with velocities. While the conception of time is not grasped operationally . . . the temporal order is confused with the spatial order and duration[1] with the path traversed. Conversely, before the temporal order has been constructed, the idea of velocity is often bound up with that of overtaking, i.e. with a purely spatial intuition involving a change in the respective positions of two moving bodies. The construction of time proper therefore begins with the correlation of velocities, be it in the case of human activity or of external motions

[1] Piaget here uses 'duration' in a different sense from that which we saw earlier. Here 'duration' refers more or less to the quantitative measurement of time intervals, i.e. 'how long something lasts.'

[PIAG6: 279].

Piaget identifies three post-sensorimotor-phase stages in the child's conception of time. The first, stage I, takes in the period of ages 1½ to 4 years. His principal finding for this stage is,

> In short, at stage I, the child must reconstruct into concepts his elementary ideas of succession and duration. Now, at that stage his constructions remain exceedingly primitive: true, they are abstracted from their particular context and generalized by the very fact of their conceptualization, but they do not yet lead to the differentiation between temporal and spatial structures. In effect, time at stage I is simply the order of succession and colligation of durations of a single series of linear events, irrespective of its own velocity or its intersections with other series with different velocities . . .
>
> Time at stage I is therefore a localized time in the double sense that it varies from one motion to the next, and that it is confused with the spatial order. It is, one might say, a time without velocities, or a time that is homogeneous only so long as all velocities are uniform. As soon, however, as actions at different velocities are introduced, the terms 'before' and 'after' lose all meaning or else preserve their purely spatial sense; simultaneity is denied; the equality of two synchronous durations ceases to make sense; the colligation of durations can no longer be performed – nor, *a fortiori*, can the measurement of time. Even the concept of age, which would seem to be based on what the child has heard from adults, is interpreted spatially, inasmuch as differences in growth-rate lead to failure in grasping the order of succession of births and the conservation of age differences. Finally, psychological time, too, may be assessed by such spatial criteria as the results of a particular action. All these findings lead us to the same conclusions, namely that the construction of homogeneous time involves the co-ordination of velocities, and that the temporal ideas prevailing before this co-ordination is achieved, must necessarily be bound up with spatial intuitions [PIAG6: 281-282].

Above I described the sensorimotor phase concept of time as a 'tool.' Stage I described by Piaget here confirms this. Earlier in this treatise we discussed Piaget's findings on 'cognizance' and his conclusion that awareness moves 'from the periphery to the center' – i.e. that cognizance begins with goals and results and 'moves' to reflection upon and refinement of object concepts. The child knows *how* to do something (throw a ball, walk on all fours, etc.) before he knows how to *describe* what he just did. The same appears to be true of objective time: the child knows how to use the concept of time before he understands the object of the concept. To use more Kantian terminology, "understanding precedes comprehension." Piaget's findings here on objective time resonate beautifully with his findings on the development of cognizance.

During stage II the child gradually begins to differentiate time from space and to draw a clearer conceptual distinction between meanings of 'before' and 'after' in their reflective application to experienced events. This is to say that what had been an intuitive scheme of 'before-and-after' becomes understood in relationship to objects.

> During the second stage the initial intuitions slowly become differentiated or articulated, either because 'before' and 'after' in time and space become differentiated from each other, or else because simultaneity becomes recognized independently of positions or velocities, or finally because duration is understood to be inversely proportional to velocity. However, the point at which intuition becomes articulated varies from one subject to the next, nor does the initial step lead to the immediate articulation of temporal relations in general. In other words, at stage II, intuitions, even if articulated, cannot yet be combined into a general grouping [PIAG6: 92].

> Grasping time is tantamount to freeing oneself from the present, to transcending space by a mobile effort, i.e. by reversible operations. To follow time along the simple and irreversible course of events is simply to live it without taking cognizance of it. To know it, on the other hand, is to retrace it in either direction. Rational time is therefore reversible, whereas empirical time is irreversible, and the former cannot embrace the latter unless this fundamental contrast is fully taken into account . . .
>
> It is characteristic of primitive thought that it treats as absolutes the particular perspectives it happens to be dwelling upon, and that it consequently fails to 'group'. This initial 'realism' is both a form of egocentrism, since it places current consciousness at the center of everything, and also a form of irreversibility, because, in it, moment succeeds moment without leading to the construction of a general flux. More precisely, egocentrism and irreversibility are one and the same thing . . . In the field of psychological time, they mean living purely in the present and assessing the past exclusively by its results . . . The operational construction of inner time calls for the correlation of one's own time not only with that of others, but also with physical time, within a reversible system that has ceased to be egocentric and is no longer bound up with current events . . .
>
> This lack is gradually made good as the child passes from stage I to stage II . . . Here progress in intuitive regulation helps to reduce the excessive deformations that spring from the irreversible centrations . . . Articulated intuition thus marks the beginning of decentration and so prepares the way for operations . . . In the case of successions it leads to the anticipation and reconstruction of the motions themselves and deflects attention from their end points. In short, intuitive decentrations introduce corrections and these, in turn, lead to certain correlations. However, as we saw, the correlation of velocity and duration does not automatically introduce the correct order of succession or vice versa [PIAG6: 283-285].

Stage II is a transition stage from the 'pre-operatory' thought of the toddler to the fully 'operational' thought of the older child. With regard to conceptual time, Piaget notes that the transition from stage II to stage III is rapid and that the child relatively quickly comes to an operational construction of time.

> From the psychological point of view, the construction of temporal 'groupings' which marks the transition from stage II to stage III is remarkable for two paradoxical facts on which we have dwelled at some length. The first is that the child succeeds in constructing one and the same system of temporal groupings in two distinct ways: sometimes he will discover successions before he is able to colligate durations, at other times he takes the opposite path; in both cases, however, he arrives at the same operational result, i.e. he learns to base successions on durations and vice versa . . . The second fact worthy of notice is the relatively short period of transition between stages II and III, i.e. the relatively quick operational construction of time . . .
>
> Now this total solution of the problem of time can be summed up in a single formula: operational time is constructed as soon as the order of successions is deduced from the colligation of durations and vice versa [PIAG6: 285-286].

As noted earlier, a deeper understanding of this issue of the child's concept of objective time can be obtained by examination of Piaget's experimental observations. We have here seen only the summary and findings, not the data that supports them. In particular, it is important to be clear that the stage III child (age around 8 years) still has only a 'concrete' conception of time, not a formal and 'logical' understanding of it. The concrete operations of his thinking process after stage III makes use of the concept of a *reversible* time without dwelling on the fundamental conflict that arises when this concrete and operational concept of time is contrasted against a formal and abstract "ontological time" which, for most people, is an 'arrow' and 'flies only in one direction.' Only poets, science fiction writers, and now, apparently, physicists ponder over 'traveling in time'; but all adults reverse time whenever we reconstruct the past. Piaget remarked elsewhere,

> Now it so happens that while classes, relations and numbers are being formed, we can see the construction, in a remarkably parallel manner, of the qualitative groupings that generate time and space. At the age of about 8, the relations of temporal order (before and after) are co-ordinated with duration (longer or shorter length of time), whereas the two systems of ideas were still independent at the intuitive level; as soon as they become joined in a single whole they engender the notion of a time common to various movements (internal and external) at different velocities. Above all, there are also constituted at the age of about 7 or 8 the qualitative operations that structure space: the spatial order of succession and the joining together of intervals or distances; conservation of lengths, areas, etc.; formation of a system of coordinates; perspectives and sections, etc. In this connection, the study of the spontaneous measurements that derives from early estimation by perceptual "transportation" and leads, at the age of 7 or 8, to the transitivity of operational equivalences (A=B, B=C, therefore A=C) and to the formation of the unit (by a synthesis of division and displacement), demonstrates in the clearest possible way how the continuous development first of perceptual and then of intuitive acquisitions leads finally to reversible operations as their necessary form of equilibrium.
>
> But it is important to note that these different logico-arithmetical or spatio-temporal groupings are as yet far from constituting a formal logic applicable to all ideas and to all reasoning. This is an essential point that must be stressed . . . In fact, the same children as reach the operations just described are usually incapable of them when they cease to manipulate objects and are invited to reason with simple verbal propositions. The operations that are involved here, then, are "concrete operations" and not yet formal ones; being constantly tied to action, they give it a logical structure, embracing also the speech accompanying it, but they by no means imply the possibility of constructing a logical discourse independently of action . . .
>
> But this is not yet all. The same "concrete" inferences, such as those leading to the conservation of the whole, to transitivity of equality (A=B=C) or of differences (A<B<C . . .), may be easily handled in the case of one particular system of ideas . . . and yet be meaningless for the same subjects in the case of another system of ideas . . . In view of this especially, it is wrong to speak of formal logic before the end of childhood. "Groupings" are still relative to the types of concrete ideas (i.e. internalized actions) that they have actually structured, but the structuring of other types of concrete ideas, which are of a more complex intuitive nature since they depend on quite different actions, requires a reconstruction of the same groupings independently of time [PIAG29: 159-161].

Thus we have the genesis of objective time. It arises from a primitive psychological fact of the form of our experience; its earliest objective beginnings lie in practical orderings of actions; its earliest cognizance is intuitive and is a tool for thinking; its earliest concepts are schemes for ordering appearances; the later development of these concepts serve *in concreto* as structuring factors that co-evolve with concepts of objects, of objective space, and objective causality. Mental operations that employ it come to endow objective time with reversibility, and it is only after one reaches the mental development stage of formal operations that ideas of ontological time are able to take on the formal thing-like characterizations such as those we reviewed in the first section of this chapter. It is not until we reach the stage of conceptualizing ideas of *noumenal* time as a thing that the borders of possible experience are transgressed. Until that point is reached, objective time is an idea that functions for understanding constituted rules of relationships among appearances and phenomenal objects.

§ 3. The Pure Intuition of Time

We cannot avoid the need to construct transcendental ideas of objective time. We require these ideas to understand the Nature of the processes of *nous* and to understand the pure form of intuitions of inner sense. Subjective time is a primitive psychological fact and is not given to us in the form of a concept, much less as an innate idea of the mind. The functioning of the processes of reflective judgment and pure Reason stand outside the formal conditions of sense in the synthesis of apprehension, and so to understand these processes we have no option but to make an objectively valid representation of a *noumenal* objective time that can be applied to the phenomenon of mind. But this idea of *noumenal* time must not contain concepts other than those necessary for the possibility of experience, and so we seek an object we will call **transcendental time**.

The fundamental basis for the objective validity of transcendental time is primitive subjective time. Human understanding understands objects in no form other than that of empirical intuitions and the transcendental schemata of sensibility. Thus our definition of transcendental time (as well as any other sort of objective time) requires in the first place an objectively valid understanding of the pure forms of intuition. In Chapter 17 we saw that for the pure intuition of space this understanding took mathematical shape in terms of a topological synthesis. In the case of the pure intuition of time our understanding is going to take mathematical shape in terms of **the synthesis of order structures**.

Orderings comprise the logical essence of that which we call determinations in time. The phenomenon of experience *in* time is the outcome and consequence of this logical essence of the pure form of intuition. The pure intuition of time is not an empirical intuition (the singular objective representation of sensibility at a moment in time) but merely a form in the synthesis of perception. The pure intuition of time is thus not an object of any possible experience but, rather, a necessary condition for the possibility of experience in the manner in which human beings are said to 'have' experience.

In Chapter 17 we reviewed Kant's characterizations of the pure intuition of time. These characterizations put in summary form are:

> In Quantity – time is a universal pure form of sensuous intuition characterized by its integrative capacity;

> In Quality – every determinate magnitude of time is only possible through limitations of a single time grounding these determinations, and this limitation is the presentational capacity of time; thus its logical Quality is the infinite *momentum*;
>
> In Relation – the *modi* of persistence, succession and coexistence are givable (*dabile*) only through a pure intuition of inner sense;
>
> In Modality – time is a necessary representation that grounds all intuitions through a capacity to order presentations in sensibility.

Newton was correct insofar as he characterized 'pure time' as *mathematical*. The Standpoint proper for our understanding of pure time (in regard to both the pure intuition of time and the idea of transcendental time) is the judicial Standpoint. In terms of our metaphysics proper, our *definition* of 'mathematical time' calls upon the *mathematical* transcendental Ideas of Rational Physics and Rational Psychology. (It is clear that the *dynamical* Ideas concern only 'how time fits in' to the system of Nature-as-a-whole, and thus do not concern pure-time-as-Object *in* Nature).

In §1.3 we saw Bergson describe 'real duration' as "the heterogeneous moments which permeate each other." In §2.2 we saw Piaget describe 'psychological duration' as "feeling of a development and of sequential directions immanent in the states of consciousness." Both of these speak, however obliquely, to Modality in the pure intuition of time, and we will start with that.

§ 3.1 Order Structure

Ideas of relationships involving ordination permeate our day-to-day lives. The relationship "son of" is one such example: Edward son of Henry VI son of Henry V son of Henry IV son of Blanche of Lancaster. Another is the numerical relationship "less than": $1 < 2 < 3 < 4 <$ etc. Mathematics deals with order relationships through the use of what are called "partial order relations." Let us look at some of the mathematical ideas that make up this concept.

We will start with an assumption, namely that somehow or other we can define or otherwise identify an aggregate of 'members' called a *set*.[2] As an illustration let us take as a set S the people in the first example above and write S = {Edward, Henry VI, Henry V, Henry IV, Blanche}. An "ordered pair" is any pair of members in a set uniquely juxtaposed. We will use a bracket notation, (a, b), to denote the ordered pair made up of member a "on the left" and b "on the right." Thus, (Edward, Henry IV) constitutes an ordered pair and we regard this pair as being 'not the same' as the ordered pair (Henry IV, Edward). It is because we regard these two pairs as

[2] Mathematics does not take it for granted that we can always define or identify a set. When Cantor was doing his work on set theory (also called 'the theory of aggregates') it was assumed that one could always make any arbitrary definition of a set. This 'obvious truth' was upset in a dramatic fashion by Bertrand Russell's discovery of what is today known as 'the Russell paradox.' In order to avoid the Russell paradox, mathematical set theory now codifies restrictions on how a set can be defined in the axioms of set theory.

being ‘not the same’ that this juxtaposition is called an ‘ordered pair’ rather than just a ‘pair.’

The idea of the set of all possible ordered pairs of members in a set is an important formal idea in mathematics. This is called the “Cartesian product” of a set and is usually written $S \times S$.[3] For our example *S*, its Cartesian product is the set composed of all 25 possible ordered pairs, i.e. (Edward, Edward), (Edward, Henry VI), . . ., (Edward, Blanche), (Henry VI, Edward), (Henry VI, Henry VI), . . ., (Blanche, Henry IV), and (Blanche, Blanche). The main reason the Cartesian product is an important idea is because it is used to define a very fundamental idea called a “binary relation on a set.” Given our Cartesian product $S \times S$ from above, suppose we pick some subset of it, $R \subset S \times S$. Because it is a subset, *R* likewise consists of members that are ordered pairs. The specific ordered pairs that make up *R* define a binary relation on *S* represented by *R*. As an example let the binary relation on *S* be the relationship “son of” from before. Then this binary relation on *S* is the set *R* = {(Edward, Henry VI), (Henry VI, Henry V), (Henry V, Henry IV), (Henry IV, Blanche)}. Note that terms such as (Henry V, Henry VI) are not members of this *R*; they do not ‘satisfy the binary relation,’ e.g. Henry V is not the son of Henry VI. In like fashion, (Edward, Edward) is not a member of *R* because Edward is not the son of Edward.

Binary relations on a set can have different important definable ‘structural’ properties. There are four such properties in particular that are important to our discussion of the pure intuition of time. The first of these is called the ‘reflexive’ property. A binary relation is reflexive if for every member *s* in *S*, the ordered pair (s, s) is a member of *R*. Otherwise the binary relation is ‘irreflexive.’ Our example *R* from above is an irreflexive binary relation. An example of a reflexive binary relation is given by the set of integers {1, 2, 3} and the relationship “less than or equal to.” The binary relation for this is {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)}.

A third important property is the ‘antisymmetric’ property. Let *a* and *b* be any two distinct members of *S* and suppose (a, b) is a member of *R*. The binary relation is antisymmetric if for any pair $a \neq b$ the inclusion of element (a, b) in *R* implies (b, a) is not a member of *R*. The example we just gave using the three integers is an antisymmetric binary relation, and so is our “royal” binary relation example involving Edward, Henry VI, etc.

Finally, a binary relation might have the ‘transitive’ property. Let *a*, *b*, and *c* be members of a set *S* and let *R* be a binary relation on *S*. *R* is said to be a transitive relation if, for any three members *a*, *b*, and *c*, the inclusion of both (a, b) and (b, c) in *R* implies that (a, c) is also a

[3] Actually, what I am describing here is a special case of the more general definition of Cartesian product. In general we can define Cartesian products for any *ordered pair of sets*. For example, if we have two sets, *S* and *P*, with M members in the set *S* and N members in the set *P*, the Cartesian product $S \times P$ is the set of all ordered pairs (s, p) where *s* is a member of *S* and *p* is a member of *P*. $S \times P$ then has M·N members. The order is important and in general the Cartesian product is not commutative, i.e. $S \times P \neq P \times S$.

member of *R*. Our previous example involving the three integers is a transitive relation. The example involving Edward, Henry VI, etc. is not a transitive relation, e.g. (Edward, Henry V) is not a member of *R* but both (Edward, Henry VI) and (Henry VI, Henry V) do belong to it. Note, however, that if our binary relation had been "descendent of" rather than "son of" then the pair (Edward, Henry V) *would* be a member of this binary relation and in fact this relation would be transitive.[4] This example illustrates that defining *specific* binary relations on a set is not a 'pure' act of mathematics inasmuch as when we apply mathematics to talk about Nature, the structure of a binary relation is tied to meaning implications. The work of "pure mathematics" (according to the school of 'formalism') involves figuring out what is apodictically true, in the most general way, about particular mathematical structures given an agreed-upon set of basic premises (called 'axioms'), and in defining new structures that increase the scope of knowledge of 'mathematical truths.'

This brings us to the important mathematical idea of *partial orders*. There are two types of mathematical partial orders of interest to us here. The first is called a "strict partial order." A binary relation on a set is a strict partial order if the relation has the irreflexive, antisymmetric, and transitive properties. Our 'descendent of' example is a strict partial order. The second type of partial order is called the "weak partial order." A binary relation is a weak partial order if it has the reflexive, antisymmetric, and transitive properties. Our "less than or equal to" example above is a binary relation that is a weak partial order. A weak partial order relation is sometimes called a "poset" by mathematicians. Partial order relations in mathematics are important enough to be given special symbols for designating relationships. Symbol usages vary from mathematician to mathematician, but one very common pair of symbols employed is ' $<$ ' for a strict partial order and ' $\leq$ ' for a weak partial order. For our 'descendant of' example we can use this symbolic convention and write 'Edward < Blanche'; for the 'less than or equal to' example, we would write '$1 \leq 3$'.

As another example of a weak partial order let us use the set of integers {1, 2, 3, 4, 5, 6} and the binary relation 'divides'. Integer *a* is said 'to divide' integer *c* if there is an integer *b* in the set such that $a \cdot b$ is equal to *c*. If *a* divides *c* we write this as $a \leq c$. Our binary relation *R* for this example specifies the following:

for 1: $1 \leq 1$, $1 \leq 2$, $1 \leq 3$, $1 \leq 4$, $1 \leq 5$, $1 \leq 6$;
for 2: $2 \leq 2$, $2 \leq 4$, $2 \leq 6$;
for 3: $3 \leq 3$, $3 \leq 6$;

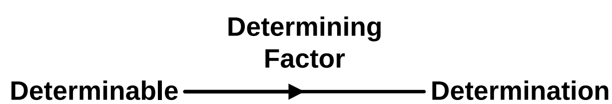
[4] Edward was the grandson of Henry V. The example we have been using is one branch in the family tree of England's King Edward III (A.D. 1312-1377). Edward, Prince of Wales, was killed at Tewkesbury in 1471 at age 17.

and: $4 \leq 4$, $5 \leq 5$, $6 \leq 6$.

The set R thus contains the 14 ordered pairs specified above. We can see that the idea of partial orders is indeed an idea with a very great scope.

Advancing these ideas yet another step, strict and weak partial orders are closely related. Suppose R is a weak partial order. Then R contains ordered pairs (s, s) for every member of the set S upon which the partial order R is defined. The subset $I_A \subset R$ consisting of all these (s, s) pairs is called the 'identity relation on R.' Now let P be a strict partial order on the same set S. The set P turns out to be a subset of R; it contains all the members of R except those (s, s) members belonging to I_A. R is therefore called the 'reflexive closure' of P. In general, if some set A is a binary relation that lacks the reflexive property, the reflexive closure of A is the smallest reflexive relation C that contains A as a subset. Here 'smallest' means "having the least possible number of members in the set necessary to contain A as a subset and still satisfy the reflexive property." Because C is reflexive, it will contain a subset I_A (as we defined this symbol above) and C will simply be the union $A \cup I_A$.

We are now ready to apply these ideas to the Modality of making a representation of the pure intuition of time. Let us recall that the ideas of Modality in our general 2LAR of representation are the determinable, the determination, and the determining factor. Symbolically, the synthetic relationship among these terms is illustrated as shown in the following figure.



The first question facing us is this: with regard to the pure intuition of time, what constitutes these three factors in the synthesis of time?

Now, Modality is the matter of *nexus*, hence the matter of the form of a manifold. When we inquire into the constitution of the above-mentioned three factors, we are inquiring into their objective character, thus putting the question to the metaphysics proper of Rational Physics. But the Standpoint pertinent to the understanding of pure time (as pure intuition) is the judicial; and the logical-judicial Ideas of Modality proclaim:

1. The representations in sensibility and the motor faculties of the Organized Being are such that the former can be joined to specific capacities for actions in the latter;

2. That which coheres with the material conditions of meanings (somatic motoregulatory

expression) is actual;

3. Necessity takes its *Realerklärung* from regulation by practical Reason, which enforces coherence in meaning.

With these Ideas, which are regulative for physical experience, we also have the transcendental-judicial Idea, which is regulative for psychological experience. The Idea is:

Unconditioned unity in apperception of all perceptions in the interrelationships of meaning.

From the Idea of judicial possibility we can see that the role of the determinable is filled by determinable relationship between sensibility and motoregulatory expression. The Idea of judicial actuality tells us that the role of the determination is filled by that which coheres with the material conditions of meanings. Neither the pure intuition of time nor that of space makes the connection between sensibility and motoregulatory expression; that task falls to reflective judgment. The determination is a sensibility that can be judged to have this coherence under the principle of formal expedience, and this condition is what is denoted by reflective judgment through the marking of a moment in time. However, coherence in meaning is not the automatic by-product of pure receptivity but, rather, must be enforced by judgmentation and reasoning, hence through the spontaneity of the Organized Being in being a source of its own representations.

The regulative principle of this Self-enforcement is the judicial psychological Idea of Modality. The pure intuition of time is a synthetic process of this spontaneity in enforcement. What we can know with objective validity of this process is deduced from its outcomes in experience. Now, the mathematical idea of *order* is well suited to stand for the matter of the form of *nexus* in sensibility. But ‘order’ in this sense must be viewed globally and in terms of the principle of unconditioned unity in apperception. No one specific and singular representation in sensibility contains this order because the mathematical idea of order minimally involves the representation of a pair. From the determinable and the determination factors identified above, we can see that this pair can be nothing other than pairs of perceptions because apperception is consciousness and representation with consciousness is perception. Therefore the fundamental ordered pairs in our mathematical representation of the pure intuition of time are comprised of pairs of perceptions. But a perception is merely that representation in sensibility which is marked at a moment in time. The mathematical elements of temporal order relationships are thus pairs of sensibilities-in-moments, which we can denote by the notation (s_i, s_j). Note that here we are not speaking merely of the restriction of sensibility to objective perception; each s_i involves affective perception as well. Because we will be dealing with such pairs $k = (s_i, s_j)$ in our discussion, it is convenient to coin a name, and so we will call such an ordered pair a **kinetic**.

Now from a purely mathematical perspective we do not require an idea of *objective* time in order to define a kinetic. Any mere set, *S*, as an aggregation 'points,' each of which we call a perception *s*, suffices to define kinetics by mere selection-in-pairs. Objective 'time' as it is commonly pictured (in terms of 'flowing' or whatever) does not enter into Modality but is, instead, introduced in Relation (form of a *nexus*). But it is also the case that no mere aggregation of kinetics, *K*, is sufficient to answer to the regulative principles of Modality without 'something else' being a part of the idea. This 'something else' is the idea of *properties* of *K*, e.g. reflexive, irreflexive, antisymmetric, transitive, or closure properties. The word *function* denotes a unity in the transcendental sense. Alongside this, we note that the mathematical idea of a binary relation is an idea of 'collecting' or 'aggregating' kinetics *according to some rule*, and the characteristics of the rule determine the specific binary relation. Thus, it is not the idea of a mere ordering of perception pairs in a kinetic in which subsists the idea of the Modality of the pure intuition of time. Rather, it is the idea of a **kinetic structure as an order structure**.

At this point in our discussion it seems prudent to re-hash what is meant by this idea of a 'structure.' I quote in full Piaget's definition of this idea:

> First of all, a structure is a totality; that is, it is a system governed by laws that apply to the system as such, and not only to one or another element in the system. The system of whole numbers is an example of a structure, since there are laws that apply to the series as such. Many different mathematical structures can be discovered in the series of whole numbers. One, for instance, is the additive group. The rules for associativity, commutativity, transitivity, and closure for addition all hold within the series of whole numbers. A second characteristic of these laws is that they are laws of transformation; they are not static characteristics. In the case of addition of whole numbers, we can transform one number into another by adding something to it. The third characteristic is that a structure is self-regulating; that is, in order to carry out these laws of transformation, we need not go outside the system to find some external element. Similarly, once a law of transformation has been applied, the result does not end up outside the system. Referring to the additive group again, when we add one whole number to another, we do not have to go outside the series of whole numbers in search of any element that is not within the series. And once we have added the two whole numbers together, our result still remains within the series. We could call this a closure, too, but it does not mean that a structure as a whole cannot relate to another structure or structures as wholes. Any structure can be a substructure in a larger system. It is very easy to see that the whole numbers are a part of a larger system, which includes fractional numbers [PIAG17: 22-23].

Once defined, a mathematical partial ordering, e.g. such as the one mathematicians call a 'lattice,' constitutes a structure in precisely the sense described above by Piaget. It therefore is proper and correct to call such a mathematical entity an order structure. However, the pure intuition of time is not itself an order structure. To say so is to presume an innate and static 'temporal template' stamped on all sensibilities. Such a rationalist presupposition is neither justified by any transcendental ground nor is such a presupposition consistent in its consequences with the facts evident in the empirical study of mental development in children. Rather, **we must regard the**

pure intuition of time as a process of synthetic order structuring by which a temporal order structure is produced.[5] This is an on-going process of structuring that persists throughout the entirety of the mental life of an Organized Being. The order structure is an open-ended system, always evolving, always undergoing transformation and retransformation in the service of the perfection of equilibrium in *conscious* presentation.

In any representation Modality adds nothing to the represented object except the relationship of this representation with the consciousness of the Organized Being. Thus, order structuring is merely the mathematical idea of the matter of *nexus* in time. Along with this merely metaphysical *nexus* we must have the other three Titles of representation as the factors in representation that explain the physical *nexus* (time Relation) and the composition of time (time Quantity and time Quality). We next turn to Relation in the pure intuition of time.

§ 3.2 The *Modi* of Relation

If anything at all can be called the 'essential logical character' of objective time, it could only be the three *modi* of time in appearances. These *modi* are *not* 'past, present, and future.' We "remember the past," we do not perceive it via receptivity; we "anticipate the future," we do not perceive it via receptivity; we "live in the present," we do not *perceive* it at all. Indeed, the empirical evidence and psychological findings we discussed earlier clearly illustrate the ideas of 'past', 'present' and 'future' are nothing more than that: simply *ideas* of objects that are pure *noumena*. No objectively valid idea of objective time can therefore be grounded in 'past', 'present' and 'future.' No objectively valid model of the pure intuition of time can represent this process *from* ideas of 'past', 'present', and 'future'; rather, these ideas are derived *from it*.

We know objects through cognition and we understand phenomena through concepts. A concept is merely a rule for the re-presentation of an intuition, and concepts are a ground for the phenomenon of memory. We also know from the principle of emergent properties that to every representation of *nous* there must be a reciprocal representation in *soma* which provides in phenomenal appearances the physical counterpart (the 'other side of the coin') to noetic representation. Neuroscience has some striking examples of this reciprocity, oftentimes obtained by studies carried out in the course of treatment for neurological problems such as epilepsy. Dr. Penfield provides us with a few examples of this:

> Recollections which are clearly derived from a patient's past memory can sometimes be forced

[5] The regulative principle to seek absolute unity of the thinking subject tells us that there can be only one temporal order structure, although under the definition of structure this structure supports the construction of substructures within, and answerable to, the structure-as-a-whole.

> upon him by the stimulating electrode. The psychical experience, thus produced, stops when the electrode is withdrawn and may repeat itself when the electrode is reapplied. Such psychical results have been obtained from stimulation of certain areas of the temporal cortex, but from no other areas of the brain.
>
> A series of brief examples may be given.
>
> First is the case of S. B. Stimulation at point 19 [a specific spot in the brain] in the first convolution of the right temporal lobe caused him to say: "There is a piano there and someone playing. I could hear the song, you know." When the point was stimulated again without warning, he said: "Someone speaking to another, and he mentioned a name, but I could not understand it . . . It was just like a dream." The point was stimulated a third time, also without warning. He then observed spontaneously, "Yes. 'Oh Marie, Oh Marie!' – Someone is singing it." When the point was stimulated a fourth time, he heard the same song and explained that it was the theme song of a certain radio program.
>
> When point 16 was stimulated, he said, while the electrode was being held in place, "Something brings back a memory. I can see Seven-Up Bottling Company . . . Harrison Bakery." He was then warned that he was being stimulated, but the electrode was not applied. He replied, "Nothing."
>
> When, in another case, that of D. F., a point on the superior surface of the right temporal lobe was stimulated within the fissure of Sylvius, the patient heard a specific popular song being played as though by an orchestra. Repeated stimulations reproduced the same music. When the electrode was kept in place, she hummed the tune, chorus, and verse, thus accompanying the music she heard.
>
> The patient, L. G., was caused to experience "something," he said, that had "happened" to him before. Stimulation at another temporal point caused him to see a man and a dog walking along a road near his home in the country.
>
> Another woman heard a voice which she did not quite understand when the first temporal convolution was stimulated initially. When the electrode was reapplied to approximately the same point, she heard a voice distinctly, calling "Jimmie, Jimmie, Jimmie" – Jimmie was the nickname of the young husband to whom she had been recently married.
>
> In these examples it seems to make little difference whether the original experience was fact, dream, or fancy; it was a single recollection that the electrode provoked, not a mixture of memories or a generalization.[6]

The common ideas of past, present, and future we owe to concepts and nothing else.

The *modi* of Relation in the pure intuition of time must speak to *properties* attending the ordering structure of time. In this consideration something Kant set down in the transcendental aesthetic in *Critique of Pure Reason* has the utmost relevance.

> All which in our cognition belongs to intuition (excepting therefore the feeling of *Lust* and *Unlust* and will, which are not cognitions at all) contain nothing but mere relationships, of places in one intuition (extension), change of places (movement), and laws in accordance with which this change is determined (moving powers[7]). But what is present in the place, or what it brings about in the things themselves besides the change of place, is not given thereby. Now through mere relationships no Thing[8] regarded as it is in itself is recognized; it is therefore right to judge that since nothing is given to us through outer sense except mere representations of relationship, this can also contain in its representation only the relationship of an object to the subject, and not that which reaches to the interior for the Object in itself. It is exactly the same in the case of inner sense. It is not merely that the representations *of outer sense* make up the proper stuff with which we occupy our mind, but

[6] Wilder Penfield, "Memory mechanisms," *A.M.A. Archives of Neurology and Psychiatry*, vol. 67, pp. 178-198, 1952.

[7] Recall that the topological synthesis of space (pure intuition of space) is a process that utilizes kinaesthetic feedback, thus is reliant upon transformations of motoregulatory expression. It is this latter that constitutes the moving powers to which Kant here refers.

[8] *Sache*.

> moreover the time in which we set these representations, which itself precedes the consciousness of them in experience and, as formal condition of the way in which we ground how we set them in the mind, already contains relationships of succession, of coexistence, and of that which is conjoint with succession (of persistence). Now that which, as representation, can precede any act of thinking is something in intuition and, if it contains nothing but relationships, it is the form of intuition, which, since it does not represent anything except insofar as something is set in the mind, can be nothing other than the way in which the mind is affected by its own activity, namely this setting of its representation, thus the way it is affected through itself, i.e. according to an inner sense of its form [KANT1a: 188-189 (B: 66-68)].

These relationships of persistence, succession, and coexistence are the *modi* of time in all our perceptions, and what we must do is understand what is required in the temporal order structuring of the pure intuition of time in order to make possible these forms of perception.

The easiest of these is persistence. That which is 'persistent in time' is represented as identity (identification) in representation. In terms of a kinetic, which is always the ordering of a pair of perceptions, this form is merely $k = (s_i, s_i)$. **The transcendental aesthetic ground of the *modus* of persistence in time is the structuring of a reflexive partial ordering**. The process of the pure intuition of time acts to construct a weak partial order structure. In set theoretic terminology, the set I_A is the logically categorical Relation put together in the synthesis of time.

The *modus* of succession in time is also easy. It is one of the most common facts of experience that we experience Nature in terms of changes, and our conceptualization of these changes is fixed in our understandings with a definite 'direction.' To use Piaget's terms, our experience contains a *practical* 'before' and 'after' which, at the beginning of life, is not cloaked in a concept (for this would be a concept of objective time) but which nonetheless is shown to us in appearance by the actions of the infant and the phenomenal display of sensorimotor schemes. Here, too, our experience 'shows us' a character of perception as sensibly continuous (which we also must expect theoretically from the synthesis in continuity, Chapter 16). It is this *practical fact* of experience that justifies and grounds our speaking of one moment in time as 'growing out of the previous moment.' In the aggregate S of perceptions we must hold that certain pairs of perceptions are *given* an explicit order in the synthesis of the pure intuition of time, and that this order is characterized by an antisymmetric property of their binary relation. It is indeed this property more than any other that tells us the synthesis of time in pure intuition is the synthesis of an order structure and not some other structure (e.g. a structure of equivalence relations). **The transcendental aesthetic ground of the *modus* of succession in time is antisymmetric structuring in the making of perceptions**. It is the logically hypothetical Relation put together in the synthesis of time.

The *modus* of coexistence in time is the one which is least obvious in terms of understanding our model of the pure intuition of time. Here James' comment about the "duration-block" character of "the specious present," and Piaget's idea of primitive "duration" are of some help to us. We can attach a real meaning to such ideas only if we regard certain particular and separable perceptions as nonetheless being 'joined' in some way from moment to moment. But because the pure intuition of time has nothing determined by the matter in perception, this 'joining' can be only an idea of form, i.e. coexistence in *nexus*, rather than in composition, through a Relation peculiar to the manifold in time. But this is precisely the same as the idea contained in the mathematical property of transitive relation. To say $a \le b$ *and* $b \le c$ *implies* $a \le c$ is to say that 'something shared' by *a* and *b* and 'something shared' by *b* and *c* is 'shared' in such a way that it is also found in common between *a* and *c*; this is nothing else but an idea of coexistence *in the form* of representation in time. This something is not persistence Relation because that pertains only to the I_A substructure of the structure of time. It is likewise not succession Relation because *a* and *c* are not *immediately continuous* in consciousness. The *modus* of coexistence in time is grounded in a *transitive* structure of time. Coexistence is succession regarded as persistence. This becomes even more apparent when we consider that for cognition to come to separate objects in the extensive magnitude of an intuition both *change* and *identity* are required (else the intuition is never analytically divisible for inferences of ideation). **The transcendental aesthetic ground of the *modus* of coexistence in time is transitive structuring in the making of the *nexus* of perceptions**. It is the disjunctive Relation put together in the synthesis of time, without which *different* successions in time could not be distinguished in experience.

If it can be permitted to speak of "chains of successions," this logical character is merely to say that different such "chains" are reciprocally determining. The transitive binary relation in mathematics is formally stated in the 'positive' manner given above, but there is a 'negative' implication to it as well. It is this: if $a \le b$ but $b \nleq c$ (that is, $b \le c$ is not contained in the partial ordering) neither the ordering $a \le c$ nor $a \nleq c$ can be immediately concluded. (Either predication is a *saltus*). The first is obvious if we look at our 'divides' partial ordering example: 2 divides 4, 4 does not divide 9, and 2 does not divide 9. But 2 divides 4, 4 does not divide 14, and 2 divides 14. With regard to cognitions of successions in time, the structure of time is not a railroad track with one line *but a network of succession pathways relating distinguishable sequences of events*. But *unity in objective time* mandates an ordering structure in which these different sequences of events can nonetheless be viewed as mere substructures in *one* totality of time. The capacity for cognition of a plurality of objects coexistent in the extensive magnitude of a singular intuition depends upon the unity under disjunction (reciprocal determination) that the transitive binary relation makes possible.

It is an easy observation to make that these three forms of Relation in the pure intuition of time are fully in accord with the general principle of Relation in the logical-judicial perspective (experience is possible only through the representation of a necessary connection of perceptions) and with the general principle of Relation in the transcendental-judicial perspective (unconditioned unity of all relationships is grounded in the *a priori* anticipation of the form of perception in time according to the *modi* of persistence, succession, and coexistence). Relation in the pure intuition of time is that capacity of the synthesis of time by which it is possible for these regulative Ideas to 'make impression upon' the synthesis of apprehension.

§ 3.3 Quantity and Aggregation in Time

The mathematical treatment of temporal *nexus* just concluded is based on the presupposition of the *Dasein* of the set of perceptions *S*. This is because the partial ordering *R* is generated from ordered pairs of perceptions (s_i, s_j) and *S* is defined to be the set in which these perceptions are members. The form of composition of *S* in mathematics has implications for the form of the partial ordering inasmuch as the definition of *R* is concerned. Recall that *R* is some defined subset of the general Cartesian product $S \times S$. But upon what is the definition of this subset to be based? This is the issue of Quantity in the pure intuition of time. As our discussion is about to show, Quantity in the pure intuition of time has some interesting implications for how we are to think of 'time' in general.

We begin with the recognition that the synthesis of composition insofar as form of composition is concerned is aggregation. We noted this long ago in this treatise (e.g. Chapter 8 §5.1 and [KANT1a: 285-286fn (B: 201-202fn)]). A 'set' is an aggregate and Quantity in representation, as form of composition, is nothing else than the representation of a set. We also recall that the transcendental schema of Quantity is 'number':

> The pure image of all magnitudes (*quantorum*) for outer sense is space, but for all objects of the senses in general, time. The pure *schema of magnitude* (*quantitatis*), however, regarded as a notion of understanding, is *number*, which is a representation integrating the successive addition of unit to (homogeneous) unit. Thus number is nothing other than the unity of the synthesis of a homogeneous intuition in general, because I beget time itself in the apprehension of the intuition [KANT1a: 274 (B: 182)].

Composition in the pure intuition of time involves 'numbering' in a sense that we are about to explain.

The basic 'unit' of the partial ordering carried out by the process of the pure intuition of time is the ordered pair of perceptions (s_i, s_j). Within the synthesis of apprehension the most basic of such a pair is composed of perceptions within James' 'duration block' of 'adjacent moments in time.' We have said that the 'next' perception 'grows out of the previous' perception according to the principle of generation, and we may 'picture this image' in diagram form as shown below.

Although we are forced, by the nature of our cognitions, to represent this in 'spatial' form, nothing more is to be interpreted from this depiction than that the two perceptions constitute a successive ordered pair with temporal ordering $s_i < s_j$. Regarded under the notion of unity, we can scarcely do better than to call this representation a **unit kinetic** of change. The symbol "<" used here denotes that this ordering of the ordered pair is irreflexive. This is 'time's arrow' in psychological duration. In mathematical terminology, s_i is said to "directly cover" s_j.

Now the usual and classical representation of the 'flow' of time is depicted by a 'timeline.' Generally this gets depicted by a figure such as the following.

Such a depiction does serve to illustrate the *modus* of succession in time. In the synthesis of aggregation we can easily 'get' from this picture the three specific unit kinetics of change $s_i < s_j$, $s_j < s_k$, and $s_k < s_l$. It is equally obvious in this picture that in the pure intuition of time the antisymmetric and transitive relations are obtainable. The first is made possible by the irreflexive ordering of the unit kinetics, and for the second we have $s_j < s_k$, and $s_k < s_l$, therefore $s_j < s_l$. In mathematical terminology, s_j is said to 'cover' s_l and s_k is said to be 'between' s_j and s_l. The term 'directly cover' we used above means there is no s 'between' the perception which is said 'to cover' and the perception said to 'be covered'.

There is another property of mathematical structure directly conveyed by this picture. Two members, call them x and y, in the set S are said to be "comparable" if one or the other of the orderings $x < y$ or $y < x$ are contained in R. If the ordering relation R is such that every x and y in S are comparable, then R is called a "total order" or **chain**. The aggregation forming the set R as depicted by the 'timeline' above is an aggregation that forms a chain.

However, there is a serious shortcoming inherent in the depiction presented above. The 'unit of aggregation' here is the unit kinetic of change, and this unit is irreflexive. Therefore, the 'timeline' depicted above supports the construction of a set R that is incapable of having the reflexive property and is thus a *strict* partial ordering. But, as we saw before, the *modus* of persistence in time requires the possibility of structuring a weak partial ordering, i.e. the possibility of the reflexive property in the set *R*. *Depiction of Quantity in the pure intuition of time by means of a 'timeline' is insufficient for the structuring of time*.

It will not at all do to just say, "Well, okay; we'll just admit pairs (*s*, *s*) in the aggregation and keep the timeline." This would be, so to speak, making a pair out of a point. There is no justification for presuming this operation. Moreover, 'time' as the idea of an object is meaningless if it is divorced from the representation of *change* and we can only regard change in real terms as perceptual differences between successive moments according to the 'timeline picture.' Third, if we introduce reflexive relation into *R* merely by 'doubling up' a point *s* to force the *Existenz* of an ordered pair (*s*, *s*), this still does not produce the *modus* of persistence in time because the 'doubled up' perception is merely one perception at one moment in time and is not suitable to convey to intuition persistence in appearance over multiple moments in time. Finally, if we try to adopt such a purely mathematical contrivance, what we are saying in *real terms* with regard to experience is that we can have a singular intuition of "the present" and this, as we have already seen, is found by empirical psychology to be in flat contradiction to our actual human experience. **A 'timeline' is simply not sufficient to represent 'time.'**

I hinted at this in the previous section when I commented that the structure of time is not a railroad track with one line but a network. A single timeline such as depicted above is like a one-line railroad. The issue facing us now is, therefore: what is necessary, in the aggregation of Quantity in the pure intuition of time, for the possibility of reflexive Relation in time?

The pure intuitions of time and space provide the forms for empirical intuition, whereas sensation constitutes the matter of intuition. Now it has been convenient to discuss the synthesis of apprehension in terms of single intuitions, but nothing in all our prior discussions has ever claimed that the synthesis of sensibility in general never contains more than one intuition in the process of being represented. All that has been claimed is that an intuition, when 'formed and ready-to-go', is marked by a moment in time, and that every such intuition 'grows out of' a 'previous' moment in time. It has been further stated that sensation and feeling make up the *materia in qua*, *materia ex qua*, and *materia circa quam* of empirical intuitions and affective perceptions. It has also been stated in this chapter that the perceptions with which we are currently dealing include the affective as well as the objective perceptions.

Let us now ask: Do we have any basis, either in Kant's theory or in experience, from which we can find a transcendental ground for deducing that 'multiple timelines' or, at least, 'multiple timeline segments' are objectively valid ideas of our model of the pure intuition of time? We first take note of a remark Kant set down in his *Prolegomena* while discussing the principles of Rational Physics (which he there called the "pure physiological first principles of natural science"):

> The first of those physiological first principles subsumes all appearances, as intuitions in space and time, under the concept of *magnitude* and is thus far a principle for the application of mathematics to experience. The second does not directly subsume the properly empirical – namely sensation, which marks the real in intuitions – under the concept of *magnitude* because sensation is no intuition *containing* space or time, although it does set the object corresponding to it in both; but still there is between reality (representation of sensation) and the null, i.e. the complete void of intuition in time, a difference which has a magnitude, to wit between every given degree of light and of darkness, between every degree of heat and of absolute cold, between every degree of weight and absolute lightness, between every degree of occupancy of space and of totally empty space, diminishing degrees can be conceived, and in the same manner as between consciousness and complete unconsciousness (psychological darkness) ever-diminishing degrees take place; hence no perception is possible that can prove an absolute absence, e.g., no psychological darkness that cannot be considered as consciousness which is only outbalanced by a stronger consciousness, and likewise in all cases of sensation [KANT2a: 100 (4: 306-307)].

"The first" to which he here refers is the principle of Axioms of Intuition. "The second" is the principle of Anticipations of Perception. Let us pay particular attention to the implication of this second principle, namely, "no perception is possible that can prove an absolute absence . . . no psychological darkness that cannot be considered as consciousness which is only outbalanced by a stronger consciousness." What does this imply if not the possibility that the synthesis of apprehension can be engaged in the synthesis of a plurality of possible objective representations?

Furthermore, the phenomenon of empirical consciousness likewise seems to implicate a plurality of representations in sensibility. Some of the flavor of this comes through when someone speaks of having "only a peripheral awareness" of something. In somewhat more scientific terms, there is the phenomenon that psychology likes to call "the cocktail party effect." This refers to the rather remarkable ability of a person to attend selectively to a single person's speech in the midst of the competing speech of many other people (as at a cocktail party, hence the name). Much psychological work has gone into trying to account for this ability, and from this work has come theories of such 'mechanisms' as 'attentional filters' and 'biased scanning.' However, there is another phenomenon that sometimes takes place in the 'cocktail party situation.' This is when a person attending to one conversation abruptly switches his attention to something else, e.g. upon hearing his own name spoken by someone else in the room.

Abilities such as this seem to be somehow related to another phenomenon of perception, namely the "figure-ground" phenomenon of attention stressed by Gestalt psychologists.

> People ordinarily organize their experience into a part to which they pay attention and a part to which they do not attend. Right now you are paying attention to the print on this page, and you are not paying attention to your breathing. The print is figure, and your breathing is part of the background. Notice that you can switch your attention between figure and ground, so that what was figure a moment ago can be ground now, and vice versa. This is illustrated [Benjafield goes on to describe a few experiments made on the effect] . . . A possible interpretation of this experiment is similar to the interpretation of subception experiments. Perhaps background material is encoded

> without the subject being aware of it, and, further, perhaps this material can affect subsequent behavior.
>
> Experiments like the ones we have just described were not favorably received by all psychologists. For one thing, these experiments appear to have serious methodological problems. How can we be sure that a stimulus presented below threshold, or a stimulus in the background, was not actually seen? If the stimulus were attended to, however briefly, then the subsequent effects of the stimulus on behaviors are not that surprising.
>
> In the past few years, however, new experiments have revived the notion that information is extensively encoded outside of awareness. Among the most influential of these are the studies of Marcel[9,10] [BENJ: 59].

We might very well take issue with whether "not paying attention to" something is equivalent to "not perceiving" something, and we might well disagree that stimulus effects on behavior "are not that surprising" if "the stimulus were attended to, however briefly," on the ground that ideas like 'attended to' and 'awareness' are not all that well defined or understood by psychology. Our vigilance should certainly be aroused in this regard when we see things like the titles of Marcel's papers (see footnote), which start off with the apparently oxymoronic phrase "conscious and unconscious perception." As we discussed in Chapter 5, empirical consciousness, Critically understood, has a degree, and 'attending to' something should not be regarded as implying that which is 'not attended to' is also unconscious.

In terms of the process of the pure intuition of time, what does it mean to say a stimulus is 'attended to briefly' and then 'attention is switched back' to some other stimulus? Even allowing for the reintroduction of *materia ex qua* into the synthesis of apprehension (via the synthesis of the reproductive imagination), the single-timeline model from above cannot logically avoid the introduction of discontinuities in experience because the irreflexive property of its compositional form prevents the representation of members of *R* necessary for the possibility of persistence in time. Shall we say this possibility is introduced by the category of substance and accident and originates from concepts? This will not do either, because the categories are homogeneous with intuition only by means of the transcendental schemata of time, and if aggregation in Quantity does not already contain the ability to represent reflexive ordering, the category loses its ground of deduction (could not be objectively valid).

But now let us suppose that the synthesis of time in apprehension does not produce a one-dimensional 'timeline' but, rather, is capable of producing a 'multi-dimensional **timescape**.' This idea is illustrated in Figure 21.3.1 below, where we now let the solid dots denote intuitions and the dashed ovals denote affective *materia* in sensibility.

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[9] Marcel, A.J., "Conscious and unconscious perception: An approach to the relations between phenomenal experience and perceptual processes," *Cognitive Psychology*, **15**, 238-300 (1983).

[10] Marcel, A.J., "Conscious and unconscious perception: Experiments on visual masking and word recognition," *Cognitive Psychology*, **15**, 197-237 (1983).

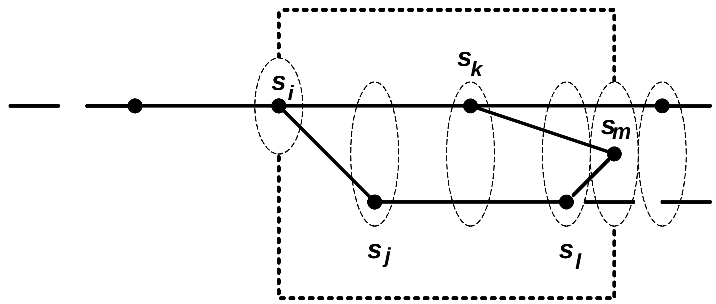

Figure 21.3.1: Illustration of a timescape. Solid dots denote intuitions. Dashed ovals denote affective perceptions (*materia* not included in the intuitions). The dotted box denotes a submanifold in time.

Giving our attention to the moments in time designated by the dotted box in the figure, we have ordered pairs (s_i, s_j), (s_i, s_k), (s_j, s_l), (s_l, s_m), and (s_k, s_m). It is easy to see that we also have two chains running from s_i to s_m, the first chain containing s_j and the second containing s_k. In mathematical terminology, perception s_m is called the **join** of s_j and s_k.

Now, the generation of s_m differs from that of every other moment shown in the figure by virtue of having two direct covers (s_k and s_l). Even though we must allow for new affects in receptivity in the synthesis of apprehension, and even though we must allow for contributions to the *materia in qua* of s_m from contributions via the reproductive imagination, it nonetheless must be the case that in the synthesis of the intuition at s_m some contribution to its contents common to both s_k *and* s_l must have survived the *Verstandes Actus* of abstraction. Because neither (s_k, s_l) nor (s_l, s_k) is a kinetic in the manifold in time, the form of aggregation in time for s_m implicates for this perception something persistent in the manifold in time. **Consequently we here have a ground for a reflexive ordering (s_m, s_m)** by virtue of s_m being the *intersect* of the two kinetics.

By itself this reflexive ordering does not implicate reflexive ordering for the other moments in time depicted in the figure. However, this is not the case when subsequently concepts re-introduce their intuitions in a synthesis of *comprehension*. Combinations in determinant judgments follow the transcendental schemata and, consequently, concepts re-introduced through the synthesis of reproductive imagination *carry with them* time-determinations which, in a multi-dimensional timescape, introduce the possibility of the synthesis in time of additional joins involving reproductions of s_i, s_j, etc. For example, the concept of s_m, owing to the schema of persistence attending (s_m, s_m), meets the condition for determination under the notion of substance. The concept of the combination (s_k, s_m) falls under the category of causality and dependency (owing to the schema of succession in time), and, furthermore, the concept of s_m in such a combination in judgment is under a notion of effect. We have commented previously, on more than one occasion, that the *Dasein* of a cause is implicated necessarily by the actuality of an effect, and that such a cause is prior in the time-order. Thus, the aggregation of a kinetic such as (s_m, s_m), which *formally* merely denotes the join of two or more timelines, supplies the necessary condition for the synthesis of comprehension to bring forth other reflexive orderings in time.

In these considerations, it is important for us to distinguish between the meaning when we say of something that it is "in time" and when we refer to the synthesis of the pure intuition of time. The pure intuition of time is a *process*, and that which it synthesizes is the pure form of inner sense, which we call subjective time. Put more bluntly, subjective time is produced in the synthesis of apprehension/comprehension. But the picture that emerges from our considerations here is not 'time as a line' (one-dimensional time) but, on the contrary, time as a timescape (multi-dimensional time) and, when all is said and done, subjective time is an order structure. Referring again to the 'physiological first principles' of Rational Physics, Kant wrote

> The above-mentioned first principles are not referred directly to appearances and their relationship, but to the possibility of experience, for which appearances constitute only the matter but not the form, i.e. they are referred to the objectively and universally valid synthetic propositions through which judgments of experience are distinguished from mere judgments of perception. This happens because: appearances, as mere intuitions *that fill part of space and time*, are subject to the concept of magnitude, which synthetically unifies the multiplicity of intuitions *a priori* according to rules; and because the real in the appearances must have a degree insofar as perception contains, beyond intuition, sensation as well, between which and nothing – i.e. the complete disappearance of sensation – a transition always occurs by diminution, insofar, that is, as sensation itself *fills no part of space and time*, but yet the transition to sensation from empty time or space is possible only in time, with the consequence that although sensations – as the quality of empirical intuition with respect to that by which a sensation differs specifically from other sensations – can never be known *a priori*, it nonetheless can be distinguished intensively from every other of the same kind; from which, then, the application of mathematics to nature with respect to sensuous intuition, through which it is given to us, is first made possible and determined [KANT2a: 102-103 (4: 308-309)].

Kant's references here, as well as elsewhere, to "filling part of time" are congruent with the transcendental proposition that has been made in this section, namely, that of the generation of a timescape by the pure intuition of time.

But before we end this discussion of Quantity in the pure intuition of time we are bound to ask: How do we know that joins such as that represented in figure 21.3.1 are a *necessary* feature of the synthesis of time? In addressing this question there are two considerations that stand out. First, the very notion of 'change' by itself has no meaning except in relationship to its complementary notion, namely 'sameness.' The irreflexive orderings denote 'change' but only the reflexive ordering (s_m, s_m) denotes 'sameness.' Secondly, the re-connection of chains in the timescape produces the *totality* of a substructure: chains are characteristics of plurality, whereas the joining of two chains unifies their individual timelines once again. Such re-unification is a requirement of the regulative Idea of Rational Psychology in transcendental-judicial perspective, i.e. "unconditioned functional unity of affective and objective perceptions in sensibility."

Furthermore, this regulative principle regulates for an ideal of time-determination through judgmentation; this ideal is that of a *complete* unity in time, the mathematical name for which is a *lattice*. This is an idea to which we will return in §4.

§ 3.4 Quality and Subjective Time

If time is a multi-dimensional timescape, why is it that we seem unable to attend to more things at once than we do? This is a very natural question to ask in light of our discussion of Quantity in the pure intuition of time. It is also a question utterly without meaning in its juxtaposition of "timescape", "attention", and the little phrase 'at once.' The question aims at understanding the Nature of our experience in time, and to sort through this issue we must examine Quality (matter of composition) in the pure intuition of time.

If one examines the graph of figure 21.3.1, long habits of thinking tend to make one regard the length of the arcs in this graph in terms of 'amount of duration' or 'length of time.' These same habits of thinking also suggest that two intuitions aligned on a common vertical in this graph 'occur at the same time.' But do ideas such as these have any real meaning? Or are they simply ideas of geometry transcendently misplaced in their application?

Let us consider the kinetics (s_i, s_j) and (s_i, s_k). In the graph the length of the arc for the former is shorter than the length of the arc of the latter, and so it is tempting to conclude that perception of s_j 'occurs before' s_k. However, such a conclusion is not objectively valid, nor is the opposite conclusion. Neither the kinetic (s_j, s_k) nor the kinetic (s_k, s_j) is presented in the time ordering depicted. In mathematical terms, intuitions at s_j and s_k are *not comparable*. The most we can say is that perceptions of both moments in time are between the moments of s_i and s_m. Subjective time is an order structure, and where no ordering is made there can be no valid comparison.

In terms of empirical apperception, the sequences of perceptions in these two temporal chains are cognitively distinct and unity in apperception is not obtained until these chains re-merge at the join s_m. Acts of judgment producing motoregulatory expression for these two chains are consciously unrelated, and it might appear to an observer (whose temporal ordering structure is different in perception) as if the acting subject is exhibiting 'more than one consciousness' or a 'secondary personality' or a 'split mind' if the appearances of these actions span many multiple *comparable* moments in the *observer's* temporal ordering.

This situation is not altered by the presentation of affective perceptions at each of these moments. This is because affective perceptions are precisely those presentations in sensibility that do not become part of cognition. Consequently, while there is an on-going *affective* unity and continuity in the synthesis of apprehension, this unity does not extend to the cognitive domain.

What meaning, then, does the idea of an 'amount of duration' have for the Organized Being with regard to subjective time? Here let us recall Piaget's "impressions of psychological duration inherent in attitudes of expectation, effort, and satisfaction" and James' observation that "tracts of time filled . . . seem longer than vacant ones of the same duration when the latter does not exceed a second or two. This . . . becomes reversed when longer times are taken." Although these two men use the word 'duration' in different ways, both remarks speak to the issue at hand. Piaget's 'psychological duration' speaks, however indirectly, to consciousness of affective perceptions bound up in a kinetic (s_1, s_2) when s_1 directly covers s_2. James' remark speaks to the chain of cognitive moments that "fill time" between some s_1 and s_n where in the ordering s_1 covers, but does not necessarily *directly* cover s_n. Both men's ideas therefore speak to a general idea of a "magnitude in duration" that rests on a concept of *intensive* magnitude in the presentation of sensations and feelings. However, both men also worked under the presupposition of an objective 'time' regarded as one-dimensional, and 'time' as timescape obviously complicates the picture of that with which we must deal.

Now, the pure intuition of time has nothing to do with sensation-in-general (sensations as the matter in an intuition and feelings as the matter in an affective perception) in representations of sensibility. All it does is provide orderings for moments in time and evolve these orderings into a structure in experience. If reflective judgment did not mark sensibility with moments in time, subjective time would have no real meaning whatsoever, nor could ideas of time ever occur to the Organized Being. Just as the pure intuition of space organizes sensibility from the receptivity of effects of motoregulatory expression (topological synthesis in extensive magnitudes), the pure intuition of time organizes sensibility from the spontaneity of acts of reflective judgment (synthesis of order structure in intensive magnitudes). A 'moment in time' is not a 'part' of time but merely a mark to be paired in an order with another mark in the pure intuition of time. An intensive magnitude has no appearance at any singular moment in time, and can only be conceptualized in terms of difference (change in content) in the sensuous 'filling' of space and time between comparable moments.

Nonetheless, there is an interesting and important relationship between the subjective timescape synthesized by the pure intuition of time and the sensible appearances of objects in intuition. In Chapter 17 we explained the pure intuition of space as a process synthesizing a topological structure. These acts come into perception at singular moments in time and, although the possible constructs were conditioned by the moment which directly covered the moment of their presentation, we provided no comment in Chapter 17 that illuminated any rule for this construction. The acts were presented in terms of the production of subsets of the *materia* of sensibility. Now, a subset $B \subseteq A$ is a representation that falls under the mathematical definition of a weak partial order (the notation $B \subset A$ denotes a strict partial order). But from whence comes an ordering making possible the topological structuring of space? The answer, of course, is: from the synthesis of the pure intuition of time. Here it is instructive to note a mathematical theorem taken from topology theory[1]:

> Let X be a partially ordered set under some ordering, $\leq$, i.e., (a) $x \leq y$ and $y \leq z$ imply $x \leq z$, (b) $x \leq y$ and $y \leq x$ imply $x = y$, and (c) $x \leq x$ for all x in X. Define $S(x)$ as a set of points y such that $y \leq x$ and define U as the set of supersets N that include $S(x)$, i.e. $N \supseteq S(x)$ and $U = \{N\}$. Then U is a neighborhood system at x and the structure $[X, U]$ is a topological space. U is called the topology of X.

What this theorem tells us is that to the extent the pure intuition of time succeeds in synthesizing a weak partial order structure, so also the pure intuition of space can succeed in synthesizing a topological structure of subjective space.[2] Neither subjective space nor subjective time 'spring into being at once and complete'; they are co-evolved as open systems and in their origination both are representations made possible through *affective* perception. Furthermore, because the form of the multi-dimensional timescape depends in part on intuitions (which require a spatial form), and because the synthesis of spatial form depends upon the form of the timescape, ***subjective space and subjective time are co-determining and not separable in real experience***. It would seem that mathematician Hermann Minkowski (1864-1909) enounced a truth more profound than he probably realized when he said, "Henceforth space by itself and time by itself are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."[3]

To these considerations we now bring in the mathematical regulative principles of Quality. In logical-judicial perspective we have: The intensive magnitude (degree) of sensation-in-general presents the complete condition for marking sensibility at a moment in time. In transcendental-judicial perspective we have the Idea of unconditioned unity in compatibility, i.e. affective and objective perception in combination make up the complete state of conscious representation. Referring again, as an illustration, to figure 21.3.1, the affective *materia* of perception overlays the structuring of the timescape like a blanket. Although no part of any objective perception, this *materia* constitutes the matter of affective perceptions, and, through change in this *materia* from

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[1] John D. Baum, *Elements of Point Set Topology*, NY: Dover Publications, 1964, pg. 23.

[2] It can also be noted in passing that the reciprocal relationship between the multi-dimensional timescape and the topological synthesis of subjective space presents us with a straightforward basis for the Gestalt psychologists' 'figure-ground' phenomenon.

[3] H. Minkowski, "Space and time," *Address delivered at the 80th Assembly of German Natural Scientists and Physicians, at Cologne*, Sept. 21, 1908.

each conscious moment to the next, presents the condition for actuality in the *Lust*-act of the adaptive *psyche* (the act of innovation in the Modality of *Lust-Kraft*), thereby constituting the empirically real in psychic expedience and in motivation in reflective judgment. This is the judicial *Realerklärung* of what it means to say that sensation and feeling 'fill time' and, from their role in motivation and psychic expedience, what it means to speak of the 'intensity of feeling and sensation.' The matter of composition in the pure intuition of time subsists in the affective relationship between the timescape and the conditions it presents to *psyche* and motivation. The ordering-in-time is also the ordination of actions, and this ordering determines the appearance of what-is-being-attended. All *concepts* of 'duration' take their Quality from this *kinesis* in the matter of perception between comparable moments.

§ 3.5 Endnote on the Pure Intuition of Time

You will have no doubt noticed that the 'matter components' in this theory of the pure intuition of time are rather less 'rigorous' in mathematical terms than are the 'formal components' (Quantity and Relation). A brief reflection shows us why this is. In its logical essence the pure intuition of time is a form-producing process. Whether we speak of 'time' in the context of subjective time or of objective time, the 'character of time' can only be a mathematical character and its 'nature' must be described and explained in detail as a partial ordering. Quantity and Relation in the pure intuition of time address this explanation.

The 'matter components' (Quality and Modality) of the pure intuition of time speak only to the reference of this process to the Organized Being's faculty of pure consciousness. The Quality of time-synthesis affirms or negates through affective perception, and the Quality of the form of inner sense therefore lies in the relationship of affective perception to *psyche* and motivation. To put it somewhat poetically, "time enters in to Reality through feelings," and this is why we give the name 'transcendental *aesthetic* of time' to the doctrine of time. The Modality of time-synthesis speaks to the relationship of inner sense to apperception. From this title we have the real context of the manifold of time as essentially an ordering structure, outside of which 'time' has no objective ground and no meaning.

§ 4. Transcendental Time

As was noted in Chapter 20 and elsewhere, when we make a theory we are making the ideas of a doctrine and it is the Nature of human intelligence that our understanding of the objects of any doctrine are and must be exhibited by means of sensuous intuition. This means that we can represent for ourselves no object except as appearances in subjective space and subjective time. This holds true for objective time as much as for any other object. Objective time is pure *noumenon*, and so all our ideas of this *noumenon* must be firmly anchored in accordance with Critical epistemology. This applies especially to any ontology of objective time. Furthermore, we cannot avoid dealing with this *noumenon* because it is only through ideas of an objective time as a *logical order* that we are able to build a doctrine for the purely intelligible aspects of mind. This is most especially the case in the doctrine of pure Reason and the causality of freedom. How shall we conceptualize objective time?

From the ancient Greeks and right up to today, objective time has been regarded as one-dimensional. Even Kant viewed it as such. There have been, however, two major themes that have played out for one-dimensional time. In his creation myth, *Timaeus*, Plato wrote:

> Now the nature of the ideal being was everlasting, but to bestow this attribute in its fullness upon a creature was impossible. Wherefore he resolved to have a moving image of eternity, and when he set in order the heaven, he made this image eternal but moving according to number, while eternity itself rests in unity, and this image we call time. For there were no days and nights and months and years before the heaven was created, but when he constructed the heaven he created them also. They are all parts of time, and the past and future are created species of time . . . These are the forms of time, which imitates eternity and revolves according to a law of number . . .
>
> Time, then, and the heaven came into being at the same instant in order that, having been created together, if ever there was to be a dissolution of them, they might be dissolved together. It was framed after the pattern of the eternal nature – that it might resemble this as far as was possible, for the pattern exists from eternity, and the created heaven has been and is and will be in all time. Such was the mind and thought of God in the creation of time [PLAT3: 1167 (37d-38c)].

St. Augustine's doctrine bears some similarity to this. It is not impossible that Augustine might have gotten his idea from Plato, although clearly Augustine put much thought into the question and, if he did come upon this idea from Plato's writings or from those of the Neo-Platonists, it is clear that Augustinian time is not the same as Platonic time.

In Plato's idea 'time' is "a moving image according to number" and, being an image, 'time exists' in the mind. But this 'moving image' is held to be the image of an unmoving and unchanging 'eternity' where everything is laid out in fixed order. Predestination would seem to be one immediate consequence of this idea. Plato, however, seems to not have been inclined to conclude predestination from this picture.

> God invented and gave us sight to the end that we might behold the courses of intelligence in the heaven, and apply them to the courses of our own intelligence which are akin to them, the unperturbed to the perturbed, and that we, learning them and partaking of the natural truth of reason, might imitate the absolutely unerring courses of God and regulate our own vagaries [PLAT3: 1175 (47b-47c)].

Still, it is difficult to see how non-predestination could be compatible with 'time as the image of eternity.' The Stoics, for example, took this predestination in nature as a first principle. "The Fates guide the man who wishes to be guided; the man who does not wish to be guided they drag along."

Present day physics does not quite know what to make of 'time.' There are two main schools of thought here, sometimes called "relationism" and "substantivalism." The relationism school of thought holds that 'time' is not a 'thing.' This view leads to a picture of 'time' very much like Plato's 'eternity.' Time is regarded as being laid out in its entirety; this picture is sometimes called "block time" and sometimes referred to as a "timescape." In this picture time does not 'flow' or 'pass' and there is no special 'place in time' that can be called 'the present.' Relationist time produces a "deterministic" picture of nature, and the idea of block time is rooted in the general theory of relativity. The 'issue with time' arises from attempts to bring the general theory of relativity into alignment with quantum mechanics. In quantum physics there is a mathematical procedure, called "canonical quantization," that is used to take classical physics' ideas over into the quantum theory. When this procedure is applied to relativity the result is an equation known as the Wheeler-DeWitt equation. What is interesting about the Wheeler-DeWitt equation is that 'time' drops out of it altogether. In effect, the equation says the universe is never-changing.

Opposed to this view are the substantivalists. The most peculiar feature of quantum mechanics is its classical non-determinism. We can predict the likelihood of each of a multitude of possible outcomes in a given physical situation, but we cannot predict what a particular actual outcome is specifically going to be before-the-fact. Block time seems to rule this out. If, on the other hand, 'time' is some kind of thing-in-itself with the property of being a continuum, then general relativity must be made 'indeterministic' (i.e. made to contain an element of randomness). The substantivalists adopt this 'time as thing-in-itself' view, and the impartial non-physicist observer could almost say that they side with Newton against Einstein. Proponents of 'string theory' in physics tend to be substantivalists.

The two different views produce entirely different theories and so even James would have to admit that the issue is 'pragmatically relevant.' Of course, neither side claims to be right in the sense of having a theory that decides the issue once and for all (so far as anything in science is ever regarded as being decided). Physics is exploring two pathways here, both known to be and admitted to be hypothetical at this point, and is expecting one or the other to eventually prevail. After all, there would seem to be no middle ground between 'time is not a thing' and 'time is a thing.' However, there *is* a middle ground, a Critical one: objective time is a *noumenon*. I would have to say (and I do say) that the relationist-substantivalist views are ripe for producing a Critical antinomy in dialectic speculation because both sides place ontology before epistemology.

§ 4.1 One-dimensional Time and Physics

One-dimensional objective time (which I will for the remainder of this sub-section abbreviate as simply 'time') usually enters the equations of physics as an independent real variable[4] serving to produce what the mathematicians call a "parametric equation." In such an equation, time serves as a 'parameter' of the equation. A 'parameter' in mathematics is

> a constant or variable that distinguishes special cases of a general mathematical expression.[5]

A parametric equation is

> an equation that determines the coordinate of points on a curve in terms of a single common variable.[6]

A simple example of this is the equation from kinematics that specifies the height *x* of a ball moving under the influence of gravity as a function of time,

$$x(t) = x_0 + vt - 0.5gt^2$$

where x_0 is the "position of the ball at $t = 0$", *v* is the "initial velocity" of the ball, and *g* is the "acceleration due to gravity" (roughly 32 feet per second per second). x_0 and *v* are known as the "initial conditions" and it is usually assumed that parameter *t* runs from 0 continuously to 'infinity.' The values taken on by *t* are called the 'domain' and the values taken on by *x* are called the 'range.' If *x* is plotted as a function of *t* the equation describes a parabola.

This equation "becomes physics" when specific objective interpretations are inferred for its variables. In the case of this example, *x* is taken to represent 'height' and *t* is taken to represent 'time.' It is at this point where 'physical characteristics' are imposed upon the parameter *t*. The most common of these is that the physical situation is described by making *t* increase monotonically (in one 'direction') and continuously (*t* is presumed to be a real number) from an initial value (usually 0). It is this presupposition that gives us a one-dimensional, unidirectional 'time.' However, there is nothing in the pure mathematics that *requires* this presupposition; the equation 'works just as well' if *t decreases* monotonically and continuously from some initial value toward 'negative infinity.' This is called "time running backwards" and is sometimes (and somewhat carelessly) described as "*x* moving backwards in time from the future into the past." Although this affronts most people's 'common sense' and 'never appears to happen', this 'time running backwards' trick is precisely what Feynman did when he proposed that the antiparticle

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[4] The term "real variable" means a variable that takes on numerical values from the set of "real numbers."
[5] David Nelson (ed.), *The Penguin Dictionary of Mathematics*, 2nd ed., London: Penguin Books, 1998.
[6] *ibid.*

known as a 'positron' should be regarded as 'an electron moving backward in time.' If almost anyone else had proposed such a crazy-sounding idea he probably couldn't have gotten it published. In the hands of Feynman (and of Schwinger, and of Tomonaga), this 'crazy idea' became the theory of quantum electrodynamics, the crown jewel of modern physics and the last important new theory constituting a 'finished chapter' in the doctrine of physics.

Time also enters physics, again as a parameter, in another and fundamentally vital role, namely in defining what is called "the time rate of change" of some physical object. Mathematicians call the mathematical expression of this by the name "derivative." Applied to our example above, and using the modern notation, credited to Leibniz and extended by Weierstrass, the time rate of change of x is defined[7]

$$\frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t - \Delta t)}{2 \cdot \Delta t} \quad .$$

We recognize this at once. It is nothing else than Newton's "method of first and last ratios of quantities" we discussed earlier. Here the quantity Δt is taken to represent a 'time interval' that can be made arbitrarily small, i.e., "taken to the limit where Δt goes to zero." If x represents 'position' then the time rate of change of x (denoted by the symbol to the left of the "=" sign) is called the 'velocity.' Taking 'the limit $\Delta t \to 0$' makes Δt an 'infinitesimal' and today this operation is defined mathematically using Weierstrass' "epsilon-delta" definition of continuity.

Other types of derivatives can also be defined. For example, the rate of change with respect to position is often called a 'gradient.' Derivatives are of fundamental importance in physics because they are used to produce 'differential equations' – a term that reflects what we saw Feynman call "local physics" earlier in this treatise. With only a few exceptions (classical thermodynamics being one of them), the fundamental equations of physics are differential equations or systems of differential equations.

So far 'physical time' as a parameter conforms to what most people hold as their 'common sense' view of 'time.' The situation becomes more interesting when the theory of relativity enters the picture. Einstein argued that the idea of "position" has only an operational meaning defined

[7] Nowadays there is a slightly different form of equation taught in the calculus to define a derivative. This alternate form need not concern us here except to note that it is a mild corruption of Newton's original form which mathematical analysis forces to give the same formal result. When a distinction must be made between the two definitions (as it usually must be in computer analysis of equations), the 'new' definition is called the 'method of one-sided differences' and the form shown above is called the 'method of central differences.' The two methods give different results in numerical analysis, and Newton's original form is preferred there.

by measurements one makes with some measuring instrument (say, a ruler), and that the idea of "time" similarly has only an operational meaning defined by measurements one makes with some kind of clock. Suppose observer A observes some event which takes place, according to his ruler, at a location (x, y, z) in space at a time t as measured by his clock. Suppose that in addition there is a second observer B who is moving relative to A in the x direction with a velocity of v relative to A. B observes the same event as A, and according to B's ruler and clock this event is measured as taking place at location (ξ, η, ζ) at time τ. How are B's measurements related to A's measurements?

According to Newton's absolute space and absolute time, this relationship is simply

$$\xi = x - vt,\ \eta = y,\ \zeta = z, \text{ and } \tau = t.$$

Einstein, however, argued that since the speed of light is a 'universal constant' c (approximately 186,000 miles per second) which is the same for every observer in uniform motion, this affects the outcome of measurements performed using rulers and clocks. Einstein derived a differential equation for relating observer B's measurements to observer A's measurements. When this equation is solved *taking into account how measurements are made*, the results turn out to be

$$\begin{aligned}
&\xi = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad \Rightarrow \quad \Delta\xi = \xi_2 - \xi_1 > \Delta x = x_2 - x_1 \\
&\eta = y \\
&\zeta = z \\
&t = \frac{\tau + v\xi/c^2}{\sqrt{1 - v^2/c^2}} \quad \Rightarrow \quad \Delta t = t_2 - t_1 > \Delta\tau = \tau_2 - \tau_1
\end{aligned}$$

The physical interpretation is this: According to observer A, observer B's ruler will appear to be *shorter* than A's ruler (in the x direction only), and observer B's clock will appear to run slower than A's clock. Put another way, lengths of objects in motion appear to *contract* and clocks appear to *slow down*. This is why, under relativity, there can be no absolute space nor absolute time in any physically meaningful sense. Neither space nor time can be *Sache*-things.

What, then, of the substantivalists' ontological position? Relativity's argument looks pretty airtight if one accepts relativity's principle that the speed of light is a universal constant. However, there are two points at which it can be attacked. First, in quantum electrodynamics the speed of light is not a constant; it is an average. Einstein's differential equation is a classical equation; it therefore, or so goes the argument, must be made over in terms of quantum mechanical operators. But secondly – and this is a much deeper subtlety – the derivatives that go into making up Einstein's equation involve limiting arguments. Newton physically justified his limiting argument – in the teeth of objections that 0/0 could not be unambiguously defined – by arguing that the limit converges upon absolute quantities. Between Newton and Einstein came Weierstrass and his 'epsilon-delta' analysis for dealing with mathematical continuity, and, as we have seen, Weierstrass' analysis simply forces the limit to agree with Newton's value. Einstein's analysis simply used the accepted mathematics of the calculus. This is all well and good provided that 'time' is 'really' a continuum. But is it? We are back once again to an ontological presupposition, and this one is at the core of the relationist-substantivalist debate.

Looking at the equations of special relativity (which are officially known as the Lorentz transformations), we find that the parameter τ is a function not only of the time parameter t but the spatial variable x as well. Modern physics accepts as fact the idea that 'space' and 'time' are, so to speak, 'joined at the hip' and the mathematical description of the relativistic universe is a 'geometry of space-time.' This geometric interpretation was first made explicit (and famous) by the mathematician Minkowski. It is therefore not out of place to mention that the idea of 'space' is also attended by a continuity issue precisely similar to that noted above for 'time.' Derivatives of functions with respect to position (e.g., df/dx) are also common in physics and, indeed, Einstein's differential equation contains both types of derivatives. Again this is fine if 'space' is a continuum. But is it? The new 'string theory' calls this ontology into question.

Some of the nastiest (and most interesting) problems in physics occur when its differential equations yield solutions where for some value of (x, y, z, t) the solution becomes infinite. Such a point in 'space-time' is called a 'singularity.' One example is the well-known "mass of the electron" problem. Relativity theory brought to light an unexpected equality between "mass" and "energy"; this is Einstein's famous $E = mc^2$ equation that almost everyone has heard of. If an electron were truly a 'point charge' (that is, a 'particle' with zero radius), it turns out that the mass of the electron should then be infinite. It is not, of course. The mass of an electron is a very tiny number. But if the electron is not a 'point charge' (has non-zero radius), why then does it not fly apart (since it carries a strong negative charge)? What holds it together? This is an example of the sorts of questions physicists have to ask themselves (and do ask themselves) when working at the edges of physical knowledge. The mass-of-the-electron problem is as yet unsolved.

Another such example involves what is known as the Schwarzschild singularity. In 1916 K. Schwarzschild obtained one of the first exact solutions for Einstein's field equations of the general relativity theory. He modeled the gravitational field equations for a single 'point mass' and found that there was a critical radius, known today as the Schwarzschild radius, where the solution became infinite. This radius is proportional to the mass and, for every object we encounter in our normal lives, this radius lies deep within the exterior boundary of the object. In this case the mathematical conditions of the solution are not met and no singularity results. For example, the Schwarzschild radius of the earth would be only 0.89 centimeter. However, if a thing were massive enough then the Schwarzschild radius could fall outside of its exterior and this would have very interesting consequences. Such a thing is called a 'black hole,' and astronomers today think they have actually identified many black holes existing in the universe, including one at the center of our own Milky Way galaxy. However, things are not entirely tidy here. For example, the 'time' required for an object to fall from rest to the singular radius turns out to be infinite; you and I need not worry about falling into a black hole because, relative to anyone's clock elsewhere in the universe, we would never reach it. Despite this, "It is assumed, however, that formation of a black hole is possible through the gravitational collapse of an ultradense star."[8] Hmm. How long does that take?[9]

Feynman had this to say about physics' various singularities:

> In addition to these particles, we have all the principles we were talking about before, the principles of symmetry, of relativity, and that things must behave quantum mechanically; and, combining that with relativity, that all conservation laws must be local.
>
> If we put all these principles together, we discover that there are too many. They are inconsistent with each other. It seems that if we take quantum mechanics, plus relativity, plus the proposition that everything has to be local, plus a number of tacit assumptions, we get inconsistency because we get infinity for various things when we calculate them, and if we get infinity how can we ever say that this agrees with nature? An example of these tacit assumptions which I mentioned, about which we are too prejudiced to understand the real significance, is such a proposition as the following. If you calculate the chance for every possibility . . . it should add up to one. We think that if you add all the alternatives you should get 100% probability. That seems reasonable, but reasonable things are where the trouble always is. Another such proposition is that the energy of something must always be positive – it cannot be negative. Another proposition which is probably added in before we get inconsistency is what is called causality, which is something like the idea that effects cannot precede their causes. Actually no one has made a model in which you disregard the proposition about the probability, or you disregard causality, which is also consistent with quantum mechanics, relativity, locality and so on. So we really do not know exactly what it is that we are assuming that gives us the difficulty producing infinities. A nice problem! However, it turns out that it is possible to sweep the infinities under the rug, by a certain crude skill, and temporarily we are able to keep on calculating [FEYN2: 155-156].

> How can we guess what to keep and what to throw away? We have all these nice principles and known facts, but we are in some kind of trouble: either we get the infinities, or we do not get enough of a description – we are missing some parts. Sometimes that means we have to throw away some idea; at least in the past it has always turned out that some deeply held idea had to be thrown away. If you throw it all away that is going a little far, and then you have not much to work with. After all, the conservation of energy looks good, and it is nice, and I do not want to throw it away. To guess what to keep and what to throw away takes considerable skill. Actually it is probably merely a

[8] H.A. Atwater, *Introduction to General Relativity*, Oxford: Pergamon Press, 1974, pg. 146.
[9] The mathematics involved with 'black hole dynamics' rapidly gets very complicated and the picture painted by the math gets stranger and stranger, especially for 'inside the black hole.' Technically speaking, there are at least two putative 'types' of black holes. The Schwarzschild black hole is one of these; the other is called the Kerr-Newman black hole. The bold inquirer can consult Chapter 32 of C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation*, San Francisco, CA: W.H. Freeman & Co., 1973.

> matter of luck, but it looks as if it takes considerable skill.
>
> Probability amplitudes are very strange, and the first thing you think is that the strange new ideas are clearly cock-eyed. Yet everything that can be deduced from the ideas of the existence of quantum mechanical probability amplitudes, strange though they are, do work, throughout the long list of strange particles, one hundred percent. Therefore I do not believe that when we find out the inner guts of the composition of the world we shall find these ideas are wrong. I think this part is right, but I am only guessing: I am telling you how I guess.
>
> On the other hand, I believe the theory that space is continuous is wrong, because we get these infinities and other difficulties, and we are left with questions on what determines the size of all the particles. I rather suspect that the simple ideas of geometry, extended down into infinitely small space, are wrong. Here, of course, I am only making a hole, and not telling you what to substitute. If I did, I should finish this lecture with a new law [FEYN2: 166-167].

Nice problems indeed! And, so far as *the fundamental problem of connecting mathematics to physics* is concerned, they are *all* 'philosophical' and ontological.

§ 4.2 The Mother Structures

Mathematics and mathematicians, at least since the ascendancy of formalism, cannot be blamed for these problems. Mathematics does no more and no less than tell us: "If A, B, and C are true then all the following things are also true: . . ." The mathematicians do not tell us, "Time is continuous." They tell us, "If you have a continuous parameter *t* then this, that, and the other are true." In mathematics "truth" means "completely free of contradiction."

Although I do not at all like their formalism style of writing (where they start by presenting the most general and abstract ideas and seldom deign to provide any examples that would make it easier for the rest of us understand what their hieroglyphs mean), I love mathematicians. I trust mathematicians. In matters of *pure* mathematics I always have faith in what the mathematicians tell me they have found, which I cannot say of any other discipline. Mathematicians do many important things that I would otherwise have to do for myself and would not do nearly so well as they. Mathematicians build systems of mathematics and, within the hypothetical contexts that condition these systems (axioms and definitions), their systems are apodictic. It is my job, not theirs, to say whether or not these conditions apply to what I am doing.

For most of us the systematic nature of mathematics is below the radar horizon. Most people encountering mathematics in high school or college see it as preparation for the 'important things' to follow, those items of knowledge upon which a professional career or a better-paying job will depend. Math majors of course see it differently – or at least those who graduate in mathematics do. But for many of us the study of mathematics more or less is seen (to the disappointment of our mathematics teachers) as an accumulation of tricks. To solve this problem complete the square; to solve that problem use integration-by-parts, etc. We focus on the mechanics of doing mathematics and seldom take the time to notice the harmony and grandeur of mathematics' basic infrastructure. Occasionally a few mathematicians do not take the time either; I do know some who have no more idea or appreciation of it than the typical engineer or physicist.

Yet infrastructure there is. Russell and Whitehead made a small survey of the mathematics literature and took from it some 'principles' upon which they built that magnificent Gothic castle called the *Principia Mathematica*. But this is not the infrastructure to which I refer. Beginning in 1939 a group of mathematicians, who for reasons best known to themselves all published under the pen name 'Nicolas Bourbaki,' came together to examine and bring to light the structural foundations common to all branches of mathematics. Between 1939 and 1998 they published more than 40 volumes of graduate-level texts in set theory, algebra, analysis, topology, etc. These writings had tremendous impact in the world of mathematics in the 1950s and 60s. The Bourbaki were, of course, formalists and bear a large measure of the blame for the writing style I so despise. For most of us the Bourbaki texts are less decipherable than the writings of the Mayans. Nonetheless, they sought out the basic foundations of pure mathematics, and they found it consisted of three basic and irreducible mathematical structures from which all the rest of mathematics is built. These are the "Bourbaki mother structures."

Jean Piaget met a Bourbaki mathematician at a conference one time.

> As you know, the aim of the Bourbaki was to find structures that were isomorphic among all the various branches of mathematics. Up until that time, these branches, such as number theory, calculus, geometry, and topology, had all been more or less distinct and unrelated. What the Bourbaki set out to do was find forms or structures that were common to all these various contents. Their procedure was a sort of regressive synthesis – starting from each structure in each branch and reducing it to its most elementary form. There was nothing a priori about it; it was the result of an inductive search and examination of mathematics as it existed. This search led to three independent structures that are not reducible one to the other. By making differentiations within each one of these structures or by combining two or more structures, all the others can be generated. For this reason the structures were called mother structures. Now the basic question for epistemology is whether these structures are natural in any sense . . . or whether they are totally artificial – simply the result of theorizing and axiomatizing [PIAG17: 24].

Now of all people, why would Piaget take an interest in the work of a group of anonymous mathematicians whose writings were not deemed suitable for teaching until graduate study in mathematics?

> A number of years ago I attended a conference outside Paris entitled "Mental structures and Mathematical Structures." This conference brought together psychologists and mathematicians for discussion of these problems. For my part, my ignorance of mathematics then was even greater than what I admit to today. On the other hand, the mathematician Dieudonné, who was representing the Bourbaki mathematicians, totally mistrusted anything that had to do with psychology. Dieudonné gave a talk in which he described the three mother structures. Then I gave a talk in which I described the structures that I had found in children's thinking, and to the great astonishment of us both we saw that there was a very direct relationship between these three mathematical structures and the three structures in children's operational thinking. We were, of course, impressed with each other, and Dieudonné went so far as to say to me: "This is the first time that I have taken psychology

seriously. It may also be the last, but at any rate it's the first" [PIAG17: 26].

What are the Bourbaki mother structures? In the interest of getting an explanation we can all understand, we'll let Piaget, the psychologist, try to tell us.

> The first is what the Bourbaki call the algebraic structure. The prototype of this structure is the mathematical notion of a group. There are all sorts of mathematical groups: the group of displacements, as found in geometry; the additive group that I have already referred to in the sense of whole numbers; and any number of others. Algebraic structures are characterized by their form of reversibility, which is inversion in the sense I described above[10] . . . The usual definition of algebraic structure as a set on which equivalence relations are defined leads to the same properties as the definition we use here (in particular: to every theory of the equivalence relations will correspond a theory of classes).
>
> The second type of structure is the order structure. This structure applies to relationships, whereas the algebraic structure applies essentially to classes and numbers. The prototype of an order structure is the lattice, and the form of reversibility characteristic of order structure is reciprocity. We can find this reciprocity of the order relationship if we look at the logic of propositions, for example . . . This is the form of reversibility that I have called reciprocity; it is not at all the same thing as inversion or negation. There is nothing negated here.[11]
>
> The third type of structure is the topological structure based on notions such as neighborhood, borders, and approaching limits. This applies not only to geometry but also to many other areas of mathematics. Now these three types of structure appear to be highly abstract. Nonetheless, in the thinking of children as young as 6 or 7 years of age we find structures resembling each of these three types [PIAG17: 25-26].

We have previously explained what a 'group' is in Chapter 17. In general an algebraic structure is a set *S* plus some number of operations on this set such that the operation exhibits closure (i.e. the operation applied to one or more members of *S* results in a member of *S*). In mathematical notation, if g_2 denotes a binary operation then g_2: $S \times S \rightarrow S$ means g_2 assigns to every ordered pair of members of *S* some member of *S*. The usual abbreviation for this basic algebraic structure is [S, g_2]. A "unary" operation g_1 simply transforms each single member of *S* to some member of *S*, and is written g_1: $S \rightarrow S$. A "ternary" operation g_3 takes every ordered triplet of members of *S* and assigns this triplet to some member of *S*; this is written g_3: $S \times S \times S \rightarrow S$. This scheme can be extended to "4-ary operations", "5-ary operations", etc. (A "binary" operation in this lexicon is a "2-ary" operation, a "ternary" operation is a "3-ary" operation, and a "unary" operation is a "1-ary" operation).

If we specify no other property than this for an operation g_n, i.e. if closure is the only thing we assert as a property of g_n acting on *S*, then [S, g_n] is called a 'groupoid.' As more properties of an operation g_2 are specified we get algebraic structures with increasingly more "structure in them," e.g. groupoid to semigroup to monoid to group as we discussed in Chapter 17. It turns out that a 3-ary operation can be regarded as the successive application of two binary operations, i.e.

[10] This is reversibility in the sense of inversion as in A - A = 0.
[11] An example of this is: A < B implies B > A.

$$g_3: S \times S \times S \to S \text{ is the same as some } g_{2a}: (g_{2b}: S \times S) \times S \to S.$$

We can similarly regard a 4-ary operation as being 'built' from three 2-ary operations, a 5-ary operation as being built from four 2-ary operations, etc. If G is a set of binary operations then this construction process can be regarded as the result of applying some logical rule of construction to G in combination with the 'algebra' of each of the binary operations in G. A unary operation can be regarded as first specifying a set $B \subset S \times S$ (to define a reflexive subset B that contains only the pairs (s, s) for each s of S) followed by some binary operation applied to B. The point of all this is that we really only need the set S and a set of binary operations G combined with some set $B \subset S \times S$ in order to produce an algebraic structure of whatever complexity. Complicated partial orderings can be applied to an algebraic structure, and this can likewise be joined by the application of a *topological* structure τ (both of which can be constructed through the application of simpler constructs). This is what is implicated in the statement "by making differentiations within each one of these structures or by combining two or more structures, all the others can be generated" in the quote above. Mathematical structures of whatever complexity can be *generated* if we have a relatively small number of fundamental capacities for generating algebraic, ordering, and topological structures.[12]

Now, Piaget does not claim that the child in, say, the developmental stage of concrete operations 'has' *a priori* a finished repertoire of such mathematical structures. His findings are that young children possess the capacity to *construct* such structures through elaboration of simpler schemes developed during the pre-operational and sensorimotor stages of development. It is this primitive capacity to which he refers when he speaks of a "logic of meanings" developed from a starting point of the child's innate "logic of actions" capacities.

> Certain recent studies on 'constitutive functions', or preoperatory functional schemes, have convinced us of the existence of a sort of logic of functions (springing from the schemes of actions) which is prior to the logic of operations (drawn from the general and reversible coordinations between actions). This preoperatory 'logic' accounts for the very general, and until now unexplained, primacy of order relations between 4 and 7 years of age, which is natural since functions are ordered dependencies and result from oriented 'applications'. And while this 'logic' ends up in a positive manner in formalizable structures, it has gaps or limitations. Psychologically, we are interested in understanding the systematic errors due to this primacy of order, such as the undifferentiation of 'longer' and 'farther', or the non-conservations caused by ordinal estimations (of levels, etc.), as opposed to extensive or metric evaluations. In a sense which is psychologically very real this preoperatory logic of constitutive functions represents only the first half of operatory

[12] In my own engineering research I frequently encounter problems not easily solvable using the 'mainstream' mathematics that works so well in a large class of problems. In these instances, I find it useful to 'make my own math' by defining new algebraic, ordering, and topological structures. These activities go by the name "set membership theory," a specialized branch of system theory which dates its beginnings back to early applications in control system work carried out in 1968.

> logic . . . and it is reversibility which allows the construction of the other half by completing the initial one-way structures [PIAG3: v].

We have already discussed Piaget's findings on coordinators, constitutive functions, compensation behaviors, etc. and will not repeat these details here. His detailed discussion of the theory of these abilities is found in *Psychology and Epistemology of Functions* [PIAG3], *Toward a Logic of Meanings* [PIAG12], and *The Development of Thought* [PIAG19] with additional details in *The Growth of Logical Thinking* [INHE] and *Learning and the Development of Cognition* [INHE2]. The underlying studies that provided the facts leading to the theory are found in various of Piaget's works already cited as well as in *The Child's Conception of Geometry* [PIAG9], *The Child's Conception of Number* [PIAG10], and *The Origin of the Idea of Chance in Children* [PIAG23]. What is important for us here is the empirical finding that mental development takes shape in the form of constructions that *build* those same structures that Bourbaki mathematicians call the mother structures.

In our model of the Organized Being we find the ground for the possibility of this construction in the functioning of judgmentation (algebraic structuring), the synthesis of the pure intuition of space (topological structuring), and the synthesis of the pure intuition of time (order structuring). However, the pure intuition of time operating alone produces an ordering structure that does not contain reversibility. Put another way, the synthesis of subjective time provides partial orderings $a \le b$ but does not provide the reciprocal partial ordering $b \ge a$. The pure intuition of time is necessary for the possibility of constructing more complex order structures but is not sufficient by itself to 'perfect the job' by synthesizing reciprocal relationships, i.e. some $b \ge a$ to 'go with' the ordering $a \le b$.[13] In other words, from the pure intuition of time we can immediately obtain the representation of a "before-and-after" structure, but not distinct concepts "before" and "after." This requires the capacity for ratio-expression in the synthesis of appetition (Chapter 20), which brings us to the question of how concepts come to be made.

§4.3 The Induction of Concepts

We have seen that a concept is made when the synthesis of re-cognition in imagination takes an undetermined intuition over into the synthesis of determining judgment. The *Dasein* of general concepts are owed to the process of reflective judgment and specifically to the inference of ideation in teleological Relation under the *momentum* of reflective subjection (Chapter 18, §5.3).

[13] Similarly, basic classification capacities which give rise to algebraic structure are provided by the process of determining judgment, but determining judgment alone is not sufficient to perfect an algebraic structure. It is because the synthesis of the pure intuition of time in receptivity produces only one-way temporal orderings (the 'arrow of time') that reversibility in childish operations is slow to conceptually develop.

What we must now do is examine the process of synthesis by which this takes place.

Unconceptualized Intuitions

Although every intuition is an objective perception, the object of an intuition is nothing more than an undetermined appearance. As far as sensibility is concerned, all that is required to represent an intuition is that some *materia in qua* of sensation be given a spatial form (topological structuring) and ordered in subjective time. But merely because sensibility presents an intuition, it does not necessarily follow that this intuition undergoes the synthesis of re-cognition in imagination to produce a concept of that intuition. An unconceptualized intuition requires nothing more than the free play of teleological reflective judgment and the synthesis of apprehension to form the merely problematic desiration of a phoronomic preference. This is the *momentum* of the presupposing judgment in the Modality of teleological reflective judgment. In teleological Quantity this raw empirical intuition is merely a by-product of a contextual implication (teleological plurality), i.e. an intentional differentiation through the production (via actions) of a topological neighborhood in the topological synthesis of the pure intuition of space. An intuition unconceptualized has no participation in a meaning implication[14], hence teleological Quality (real tendency, real repugnance, or real significance) does not touch it, nor does teleological Relation (reflective subjection, reflective expectation, or reflective transferal).[15]

The main business of teleological reflective judgment, insofar as it is concerned with sensibility at all, is action implication (the factory of meanings and the loom of Nature as a system). The teleological *momentum* of the presupposing judgment is the repetition coordinator (a constitutive function of judgment), and so the unconceptualized intuition emerges from a scheme of circular reaction. The most primitive of these belong to the 'pre-logic' (as Piaget puts it) of sensorimotor intelligence, and successful actions produce equilibrium in the form of a cycle, such

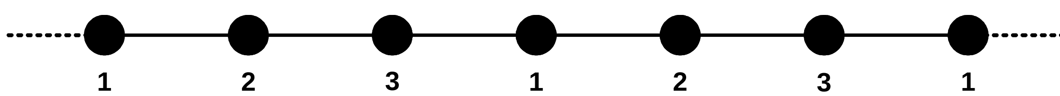

Figure 21.4.1: Illustration of a sequence of intuitions in a cycle in equilibrium. The numbers identify intuitions that contain the same *materia in qua* of representation.

[14] To say the undetermined object of appearance represented by an unconceptualized intuition has any meaning for the Organized Being is a contradiction in terms. An object is *made symbolic* of a meaning, but this requires a determinant judgment of Modality, therefore conceptualization of the intuition. See Chapter 10 §1.

[15] Teleological reflective judgment is the form of reflective judgment in general, and as such it works from affective, not objective, perceptions to form the manifold of desiration. Intuitions are merely by-products of acts of reflective judgment.

as illustrated by Figure 21.4.1. (For simplicity, the figure represents only a single chain and the affective perceptions are not shown). Let us examine the genesis of this timeline as it might appear in Stage 1 or Stage 2 of sensorimotor intelligence. Absent of object concept, the Quantity of aesthetical reflective judgment providing the ‘energetic’ for the scheme of action is the subjectively particular (sense of continuity in the synthesis of apprehension). Because we have no concept, there is no question of a free play of determining judgment and imagination in this situation, and so the Quality of aesthetical judgment here is the subjectively affirmative (pleasure). In aesthetical Relation, this action is subjectively valued solely for its subjective expedience for the Subject’s general state of happiness, hence is the subjectively disjunctive (reciprocal interest; it is an Ideal-desire, corresponding to an appetite of instinct). In Modality it is subjectively assertoric (feeling of liking).

Now, when we examine the direct covers in subjective time for this case we have

(1, 2); (2, 3); (3, 1); (1, 2); etc.

and when we examine the next (non-direct) covers we find

(1, 3); (1, 1’); (1,2’),	where 1’ denotes that the ordering is not reflexive;
(2, 1’); (2, 2’); (2,3’),	where 1’ denotes $2 < 1$, i.e. the first 2, second 1 in the sequence;
(3, 2’); (3, 3’); (3, 1’’);	
etc.	

The first question we should now ask is: Do the cover kinetics (1, 1’), (2, 2’), (3, 3’) etc. provide the ground for a reflexive time-determination (i.e. $1 \leq 1'$ rather than merely $1 < 1'$)? The answer is: No, not at all. Figure 21.4.1 illustrates a mere chain, and to determine that (1, 1’) is reflexive implies some evaluation of the *sensational contents* of 1 and 1’. But the pure intuition of time has nothing to do with sensations contained in an intuition other than to collect and combine them from the *materia ex qua* of sensibility. Nor are the pure intuitions judgments. Metaphorically, “time does not know 1 and 1’ are ‘the same’.” Logically, if $1 \leq 1'$ then it would follow that $3 < 1' = 3 < 1$, which violates the antisymmetry property of the partial ordering of the pure intuition of time because we also have $1 < 3$. This can only be logically resolved by setting $1 \leq 3$ and $3 \leq 1$ which implies $1 \subseteq 3$ and $3 \subseteq 1$ (spatial determinations that are obviously incorrect; intuitions 1 and 3 are distinct). Therefore a logical contradiction arises and we must conclude that (1, 1’) etc. cannot be the ground for a reflexive ordering in time. A reflexive ordering can only result from a purely *formal* consideration – that is to say, from the form of the *manifold* in subjective time. This is why a join is required for its possibility, and this is why we must regard subjective time as a multi-dimensional timescape.

The Re-cognition of an *Unsache*-thing

However, the situation depicted in Figure 21.4.1 is a cycle in equilibrium, and so there now enters in to our consideration the judgment of satisfaction that attends the establishment of such a cycle. All that we require is the merest innovation in the topological structuring of space to produce the set {1, 2, 3} and the *Verstandes Actus* of apprehension can "present the embodiment" of the equilibrated cycle. This makes possible a very important teleological judgment: Objective implication in Quantity; real significance in Quality; reflective subjection in Relation; the demanding judgment in Modality. Objective implication is the presentation of a syncretic *Obs.OS* which, in terms of desiration, is the intentionally integrating judgment, the judgment of teleological universality. Real significance, which in behavioral terms is type-γ compensation behavior, is the judgment of a synthetic coalition of the presentation of sensibility for an Object. Reflective subjection makes the inference of ideation, *which is a triggering judgment for the synthesis of re-cognition in imagination*. The demanding judgment is the teleological judgment that 'ignites' (so to speak) the free play of imagination and determining judgment.

At this moment in time we find no *a priori* ground for presupposing a change in the combination in aesthetical judgment *merely* because the pure intuition of space has re-structured an intuition. Indeed, we should rather expect aesthetical reflective judgment to maintain its judgment of aesthetical continuity in sensibility. But there is now an important innovation in the timescape, which is illustrated in Figure 21.4.2. This is the branching off from some moment to produce the intuition denoted 1-2-3 shown in the figure. (This has, for convenience, been illustrated as a branch from an intuition 1, but the branch can just as well be placed at a 2 or a 3).

The synthesized intuition {1-2-3} becomes, via the inference of ideation through teleological judgment, a concept through the synthesis of re-cognition in imagination. However, this act alone is not completely sufficient to 'fix' this representation as a concept. To accomplish this, there also

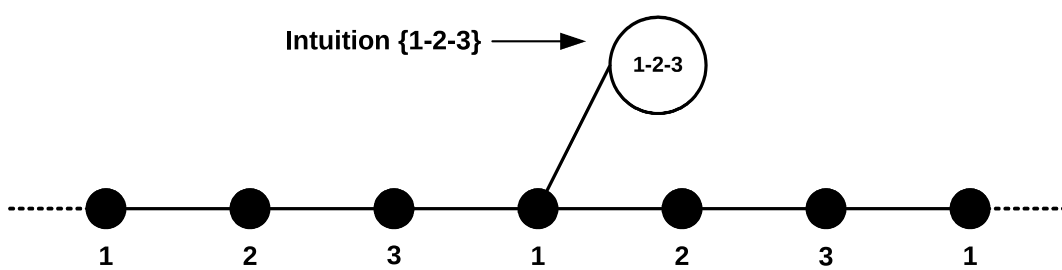

Figure 21.4.2: Branch point in subjective time in the synthesis of an intuition of an *Unsache*-thing.

is required the re-presentation of this nascent concept back into the synthesis of comprehension via the synthesis of reproductive imagination. This is because teleological inferences of ideation, while able to trigger the synthesis of re-cognition, still have nothing to do with objects, and teleological judgment does not command determining judgment. Intuition {1-2-3} has been judged *logically* expedient, but it is not yet *subjectively* expedient. That judgment requires the process of aesthetical reflective judgment.[16]

Figure 21.4.3 illustrates a possible timescape for the synthesis of comprehension following the reproduction of {1-2-3} in sensibility. The join at intuition U represents the comprehension point in the timescape. The aesthetical reflective judgment that marks this moment in time is in Quantity the subjectively singular (sense of culmination, marking an equilibrium in the free play of imagination and understanding); in Quality the judgment is feeling of beauty (the subjectively infinite feeling of satisfaction); in Relation the judgment is subjectively hypothetical (transeunt interest as the sense of *Unsache*-desire); in Modality it is subjectively assertoric (feeling of presentment). No change in the *momenta* of teleological judgment enters in here.

The time-orderings at moment U are ({1-2-3}, U) and ({1-2-3}', U), which satisfies the condition for a reflexive ordering (U, U). The concept of U, which will be the product of the synthesis of re-cognition in imagination, now connects with the concept of the intuitions {1-2-3} in the substance-accident Relation in determining judgment, and the scheme of 1-2-3 is now conceptualized as the concept of a phenomenon (Object of the *Unsache*-thing of {1-2-3}).

So far as pure forms are concerned, both in the timescape and in the topological structure of subjective space, there is nothing in figure 21.4.3 that would now prevent an additional synthesis giving rise to concepts of intuitions 1, 2, or 3 individually. However, making such a synthesis is

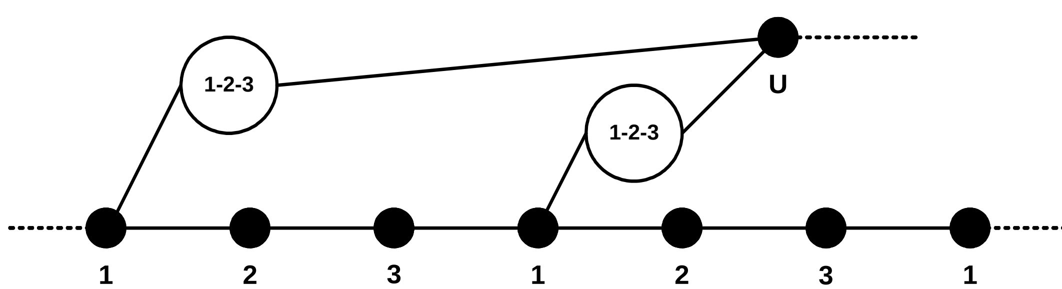

Figure 21.4.3: Possible timescape in the synthesis of the concept of an *Unsache*-thing. Branch points from the chain of intuitions in the equilibrium cycle are chosen arbitrarily. In the text, the right-most intuition 1-2-3 is called {1-2-3}'. Note that it is not necessary that {1-2-3} and {1-2-3}' contain precisely the same sensational matter since any such differences as may exist are removed in U by the *Verstandes Actus* of abstraction in the synthesis of comprehension.

[16] Recall that in reflective judgment the synthesis of continuity in *objectivity* falls to composition in reflective judgment, i.e. to aesthetical Relation and Modality (figure 16.6.1).

not automatic from mere form. A sensible condition is necessary, and that is what we discuss next.

The Differentiation of 'Before' and 'After'

The discursive activity following upon the topological structuring of intuition {1-2-3} had its origin in a teleological judgment of logical expedience. But what was the expedience for pure practical Reason? Teleological judgment does not determine the employment of the process of determining judgment, nor is determining judgment commanded by imagination. The participation of determining judgment in the process just described must be commanded by the power of Reason, and this through the ratio-expression of pure speculative Reason.

In Stage 1 or 2 of sensorimotor intelligence the sequence denoted above as intuitions {1-2-3} are effects in sensibility from the data of the senses, including kinaesthetic feedback from the Subject's actions. At these stages of mental development the practical appetite of a practical sensorimotor scheme can only be an appetite of instinct and, in its first application, an instinct is an innate practical rule unconditioned by anything in practical Reason's manifold of rules. Now, in the diagrams illustrating the syntheses above we depicted the intuitions at the various moments in time as being identical, i.e. as empirical intuitions containing no differences in the manifold of their sensational *materia in qua*. This is obviously a simplification and we have no *a priori* basis for presuming perfect identity of sensation. This was not an issue in the structuring of {1-2-3} above because the act of abstraction removes inexpedient differences in the formation of a sensuous intuition. However, such differences in the individual moments 1, 2, or 3 *are* presented to motoregulatory expression and to practical Reason in the manifold of Desire because in this manifold we have no *Verstandes Actus* of abstraction. Thus even in simple schemes of circular reaction there is work to be done for practical judgment and the synthesis of appetition in order to practically perfect the execution of the scheme. In figure 21.4.1 the internal differences in the intuitions at different moments in time do not rise to the degree of producing a major accommodation in the equilibrium of the scheme, but this is merely to say that type α compensations 'fine tune' the rule of the action if the degree of disturbance is insufficient to frustrate the equilibrium of the cycle.

> In general we speak of regulations when the reaction *A*', of an action *A*, is modified by the original action, i.e., there is a secondary effect of *A* on the new development *A*'. The regulation can then be seen as a correction by *A* (negative feedback) or reinforcement (positive feedback). However, we must specify the varieties of disturbances and underline the fact that the inverse relationship is not true; that is, all disturbances do not create regulations (hence equilibrations).

> We must distinguish two important categories of disturbances. The first include those which are opposed to accommodations: resistance of objects, obstacles to reciprocal assimilations of schemes or subsystems, etc. In short, these are the reasons for failures or errors of which the subject becomes more or less aware; the corresponding regulations include negative feedback. The second category of disturbances, the source of nonbalance, consists of gaps which leave requirements unfulfilled and are expressed by the insufficiency of a scheme. But it is worth stressing – and this is essential – that all gaps do not constitute disturbances . . . On the other hand, a gap becomes a disturbance when it indicates the absence of an object, the lack of conditions necessary to accomplish an action, or want of knowledge that is indispensable in solving a problem. The gap, functioning as a disturbance, is therefore always defined by an already activated scheme of assimilation, and the corresponding regulation then includes a positive feedback which prolongs the assimilating activity of this scheme.
>
> Although it is important to remember that any regulation is a reaction to a disturbance, the opposite can be verified only partially. We cannot speak of regulation (a) when the disturbance simply creates a repetition of the action with no modification and with the illusory hope of improvement (as is often the case with the child); (b) when the obstacle leads to the end of the action; or (c) when the subject, interested by an unexpected aspect of the disturbance, engages his activity in another direction. It is evident that in these cases we cannot speak of a modification of *A*' caused by *A*'s reaction to *A*', and in the absence of this regulation there is no reequilibration. In other words, if a regulation is to occur, a regulator must intervene [PIAG19: 18-19].

The synthesis of the intuition {1-2-3} is the cognitive counterpart to a practical regulation of the Organized Being's actions in the face of 'minor disturbances' in perception. {1-2-3} is expedient because it is the perception of the assimilated structure of the scheme. Ratio-expression serves the categorical imperative of practical Reason here by regulating *indirectly* to establish the form of a rule in the practical manifold of rules.

Within this noetic action {1-2-3} serves as a kind of 'judicial comparate' in Reason's regulation of actions – so to speak, it is a kind of 'this is what I want' representation. The re-cognition of the singular intuitions 1, 2, or 3 is not necessary to accomplish the regulatory task and so no expedience for practical Reason is served by conceptualizing them. Matters are different if we have a sensuous disturbance of a degree sufficient to require accommodation in the manifold of practical rules in order to re-equilibrate the cycle. A simplified example is illustrated in Figure 21.4.4. Here we model a sensuous disturbance in intuition 2" leading to a disturbance in

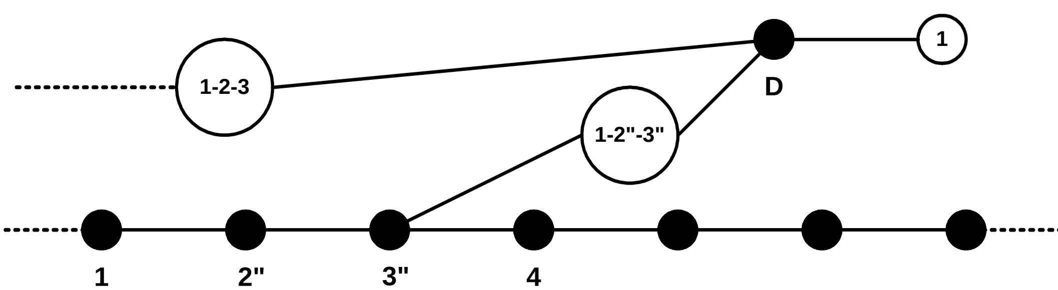

Figure 21.4.4: Model of a disturbance requiring accommodation of an action scheme. 2" and 3" are intuitions similar to 2 and 3 but containing disturbing sensuous *materia* leading to a disruption 4 in the perception of the cycle. {1-2"-3"} undergoes a synthesis such as described earlier and {1} at the upper right-hand corner of the figure likewise undergoes the synthesis of re-cognition.

3" and producing a possible disruption chain 4 in the cycle. This makes intuition {1-2"-3"} "interesting" to the Organized Being in both composition and connection in reflective judgment (and, therefore, it undergoes the synthesis of re-cognition and the subsequent synthesis of comprehension similar to what was described above). Intuition D can be regarded as the intuitive awareness of the disturbance, and its importance in this example is that abstraction can extract the intuition {1} from the synthesis of D and likewise 'feed' this intuition into the synthesis of re-cognition. With all three of these intuitions taken into concept form, the process of determining judgment now has sufficient information (from the prior combinations under the transcendental schemata of time) to produce distinct concepts (by means of the free play of imagination and determining judgment in understanding), including the connection in the manifold of concepts of series (1, {2, 3}) and (1, {2", 3"}). Thus it is, as Piaget has claimed, the phenomenon of disturbance that constitutes a necessary condition for adaptation, which always aims to restore the equilibrium between assimilation and accommodation. The *conceptual* representation of the series, e.g. (1, {2, 3}), is the genesis of a concept structure that can be characterized as representing knowledge of a 'before' and an 'after.'

All these examples are, of course, highly simplified. What must be especially stressed here is the primary role that reflective judgments play in regard to organizing concept structure in determining judgment. At the basis of all of this is, of course, practical Reason and the synthesis of appetition we discussed in Chapter 20. As an empirical illustration of the ideas presented here, let us review some of Piaget's observations on the development of the sucking reflex.

> *Observation 1.* – From birth sucking-like movements may be observed: impulsive movement and protrusion of the lips accompanied by displacements of the tongue, while the arms engage in unruly and more or less rhythmical gestures and the head moves laterally, etc.
>
> As soon as the hands rub the lips the sucking reflex is released. The child sucks his fingers for a moment but of course does not know either how to keep them in his mouth or pursue them with his lips . . .
>
> A few hours after birth, the first nippleful of collostrum. It is known how greatly children differ from each other with respect to adaptation to this first meal. For some children like Lucienne and Laurent, contact of the lips and probably the tongue with the nipple suffices to produce sucking and swallowing. Other children, such as Jacqueline, have slower coördination: the child lets go of the breast every moment without taking it back again by himself or applying himself to it as vigorously as when the nipple is placed in his mouth. There are some children, finally, who need real forcing: holding their head, forcibly putting the nipple between the lips and in contact with the tongue, etc.
>
> *Observation 2* – The day after birth Laurent seized the nipple with his lips without having to have it held in his mouth. He immediately seeks the breast when it escapes him as the result of some movement . . .
>
> The same day the beginning of a sort of reflex searching may be observed in Laurent, which will develop on the following days and which probably constitutes the functional equivalent of the groping characteristics of the later stages (acquisition of habits and empirical intelligence). Laurent is lying on his back with his mouth open, his lips and tongue moving slightly in imitation of the mechanism of sucking, and his head moving from left to right and back again, as though seeking an

> object. These gestures are either silent or interrupted by grunts with an expression of impatience and of hunger.
>
> *Observation 3* – The third day Laurent makes new progress in his adjustment to the breast. All he needs in order to grope with open mouth toward final success is to have touched the breast or the surrounding teguments with his lips. But he hunts on the wrong side as well as the right side, that is to say, the side where contact has been made.
>
> *Observation 5* – As soon as his cheek comes in contact with the breast, Laurent at 0;0 (12) applies himself to seeking until he finds drink. His search also takes its bearings; immediately from the correct side, that is to say, the side where he experienced contact.
>
> At 0;0 (20) he bites the breast which is given to him, 5 cm. from the nipple. For a moment he sucks the skin which he then lets go in order to move his mouth about 2 cm. As soon as he begins sucking again he stops. In one of his attempts he touches the nipple with the outside of his lips and he does not recognize it. But, when his search subsequently leads him accidentally to touch the nipple with the mucosa of the upper lip (his mouth being wide open), he at once adjusts his lips and begins to suck [PIAG1: 25-26].

These observations demonstrate early learning behaviors and clearly implicate the formation of concepts (in the sense of this term as used in this treatise, rather than in Piaget's sense of the word). They are presented here, first, to make clear that the diagrams shown earlier are greatly simplified depictions of the phenomenon as exhibited by human beings but, second, to demonstrate that this learning behavior is possible prior to the formation of distinct object concepts – and thus must be put against affective perception and reflective judgment in the synthesis of practical (rational) acts. Ratio-expression and thinking serve *practical* Reason and its synthesis of appetition. Reflective judgment's role is in this is in organizing thinking, which befits the task of teleological judgment in making a system of Nature. The synthesis of subjective space (topological structure) and multi-dimensional subjective time are necessary capacities for the possibility of these observable phenomena in human infants. There is also a third purpose in presenting the observations given above. This is to guard against a natural tendency of thought to impute a too-instantaneous character to the structural developments described above. How great the extensive magnitude of the timescape or the topological neighborhoods being constructed must become before they can support the development of cognition via the process described above is a matter for empirical determination, not for Critical Philosophy and much less for non-Critical ontological speculation.

§ 4.4 Continuity in Subjective Time

We have seen that objectively valid ideas of temporal orderings that are reflexive, antisymmetric, and transitive requires of the pure intuition of time that this process produce a subjective time in the form of a multi-dimensional timescape. To use Piagetian terminology, the pure intuition of time is the "constitutive function" of temporal ordering. We have likewise seen that the interplay of the structuring of subjective space and subjective time is part of this picture and that, indeed, the two forms of pure intuition, the forms of outer and inner sense, are in a relationship of reciprocity in the development of cognitive structure.

Of necessity our illustrations must appear to us in 'spatial' and 'temporal' forms, for as human beings this is the Nature of our powers of perception and cognition. Just as the only way to explain the primitive idea of representation is to *make* a representation, so also the only way to explain an idea of 'time' or 'space' is to make a representation *in* time and space. Our task, in order that these representations not make a vicious circle, is to firmly anchor ideas of *objective* time and space on the first principles of Critical epistemology.

The first rule for any idea of objective time is that this Object can never be otherwise than an mathematical Object.[1] Time *per se* is not an object of any possible sensuous experience but, rather, is pure *noumenon*. The objective validity of any idea of objective time rests on regarding time as quintessentially an ordering structure. It cannot be endowed with causative powers (*Kraft*) for it is not a substance in Nature.[2] It is, rather, part and parcel of the substratum we call Nature, a transcendental Object behind *accidents of appearances* in Nature. 'Absolute time' exists (in the *Dasein* sense of that word) only as a capacity of the powers of sensible representation in an Organized Being. All our illustrations above are representations *in the representation of* subjective space-time, which is substratum for ideas of temporal representation as a *limitation* on an idea of time-in-Nature. We can know in experience "that which 'fills' time" but not 'time regarded as it is in itself" (time as *Ding an sich selbst*).

This is why "clock time" in physics can only be relative. When we 'measure time' using a clock, all we are doing is establishing Relations of community in our objective perceptions between the appearances of one object (the clock) and some other phenomenal object (that which is having its 'duration' determined through the relating of its appearances to those of the clock object). If all we are trying to do is determine a series of successive appearances we may well use a one-dimensional model of objective time because here we do nothing more than establish a series of appearances as kinetics, e.g. (a, b), (b, c), etc. where the left element of these ordered pairs is the direct cover of the right element. But where we speak of *mathematical* co-determinations of ideas for phenomena, here we are speaking of a *reflexive* temporal kinetic (e.g. the 'position of a particle' in objective space at some $x(t)$ and its velocity – time rate of change of position – $v(t)$ at the same 'instant of time' t). A reflexive temporal ordering has no objective

[1] Similar remarks to this and the following discussion apply to ideas of objective space.
[2] It is a contradiction in terms to say that 'time' is a persistent in Nature, and if we make 'time' into a substance, this is precisely what we are predicating because we are saying "time is persistent in time." What phrase could pass our lips that is more meaningless than that?

validity in representation on a one-dimensional subjective 'time line' and must, instead, be represented in terms of a multi-dimensional timescape. Objects *x*, *v*, and clock come to a join.

This brings us back to an issue that was raised in Chapter 20, namely that of how we are to model the processes of Reason and of reflective judgment when these processes *must* be regarded as objectively standing outside the determination of the pure intuition of time. We have no option but to represent these processes of *nous* through sensible exhibitions, thus we must find objectively valid appearances of their representations and these will invariably require some accompanying representation of an objective *idea* of some form of time as a *logical order*. This idea is what I will call **free time** to denote a mere Object with none other than a *functional* role of uniting our representations of the mental anatomy and physiology of *nous*. The idea of free time is an idea represented as a timescape. But before taking up this discussion, we must first visit an idea that is quite essential for a model of free time, namely the idea of *continuity* in time.

The first inquiry we must make of this idea is: Whether or not 'continuity in time' is an idea with any *real* basis of objective validity whatsoever, and, if it does have such validity, whether there are boundaries or limitations to this validity, and, if so, what these must be. As these are transcendental questions, the foremost rule of evaluation must be, as always, *necessity for the possibility of experience*.

Subjective Time is Objectively Continuous: The Empirical Evidence

To say that subjective time is objectively continuous means that succession in perception allows no break or gap. We have two aspects to consider here, one psychological and experiential, the other mathematical. In making these considerations it is in both cases important to bear in mind that when it is said that continuity in subjective time means continuity in perception, we are speaking of affective as well as objective perception and not objective perception alone. It is *time regarded as an Object* that we are saying is continuous, and not that succession in appearances of objects is temporally continuous.

It might be assumed that the strongest empirical argument for *discontinuity* in the idea of subjective time is the phenomenon of sleep. "I have been asleep" is the conclusion we draw when we are watching a television show and the next thing we know is that the news is on and we do not remember seeing how the TV show ended. Many people regard sleep as a condition in which we withdraw from reality for an interval of time, an interlude in which our awareness is suspended. If we carry any cognitions at all across that often rather fuzzy boundary we set down between 'being asleep' and 'being awake', we call those cognitions a 'dream.'

Sleep *per se* is not an appearance and we do not actually ‘experience sleep.’ What we experience is an apparent break in our perceptions of the outer world marked by discontinuity in the appearances of objects. We place these appearances on two ‘sides’ – one ‘before falling asleep’ and one ‘after waking up.’ Regarded in this way, sleep is the object of our idea of ‘what happened in between’ the one side and the other. Were it not for the fact that each of us has had the experience of seeing other people sleeping, we would not know what to make of ‘sleep’ to any greater detail than this.

But is sleep the cessation of perception or is its essential character merely a lowered degree of receptivity? Or is it something else? The phenomenon of sleep is indeed one of the vexing puzzles for neuroscience and for psychology in general. It is even something of a puzzle for evolution theory since it seems incredible it could be advantageous to the survival of a species that a prolonged lapse of awareness should occur at night, when many predators are awake and on the hunt for their next meal. What do we know about ‘sleep’?

> Sleep, fainting, coma, epilepsy, and other “unconscious” conditions are apt to break in upon and occupy large durations of what we nevertheless consider the mental history of a single man. And, the fact of interruption being admitted, is it not possible that it may exist where we do not suspect it, and even perhaps in an incessant and fine-grained form?
>
> This might happen, and yet the subject himself may never know it. We often take ether and have operations performed without a suspicion that our consciousness has suffered a breach. The two ends join each other smoothly over the gap: and only the sight of our wound assures us that we must have been living through a time which for our immediate consciousness was non-existent. Even in sleep this sometimes happens: we think we have had no nap, and it takes the clock to assure us that we are wrong. We thus may live through a real and outward time, a time known to the psychologist who studies us, and yet not *feel* the time, or infer it from any inward sign. The question is, how often does this happen? Is consciousness really discontinuous, incessantly interrupted and recommencing (from the psychologist’s point of view)? and does it only seem continuous to itself by an illusion analogous to that of the zoetrope? Or is it at most times as continuous outwardly as it inwardly seems?
>
> It must be confessed that we can give no rigorous answer to this question [JAME2: 130-131].

We can see here easily enough that James has slipped into a presupposition that ‘time’ is something to be viewed as an external and independent denizen of Reality. This is a common and prevailing ontological prejudice that pretty nearly everyone uncritically shares. For James the issue is not so much one of the continuity of time (which he presupposes) but the continuity of consciousness and experience (which he does not merely presume). But here is precisely the point where we must be careful to make distinction between our ideas of objective time as *noumenon* and our ideas by which we model subjective time. Our ideas of objective time have their origin in understanding from the transcendental aesthetic of subjective time, and it is with subjective time that our discussion is presently involved. James go on to say the following.

> On being suddenly awakened from a sleep, however profound, we always catch ourselves in the

> middle of a dream. Common dreams are often remembered for a few minutes after waking, and then irretrievably lost.
>
> Frequently, when awake and absent-minded, we are visited by thoughts and images which the next instant we cannot recall.
>
> Our insensibility to habitual noises, etc., whilst awake, proves that we can neglect to attend to that which we nevertheless feel. Similarly in sleep, we grow inured, and sleep soundly in the presence of sensations of sound, cold, contact, etc., which at first prevented our complete repose. We have learned to neglect them whilst asleep as we should whilst awake. The mere *sense-impressions* are the same when the sleep is deep as when it is light; the difference must lie in a *judgment* on the part of the apparently slumbering mind that they are not worth noticing.
>
> This discrimination is equally shown by nurses of the sick and mothers of infants, who will sleep through much noise of an irrelevant sort, but waken at the slightest stirring of the patient or the babe. This last fact shows the *sense-organ* to be pervious for sounds.
>
> Many people have a remarkable faculty of registering when asleep the flight of time. They will habitually wake up at the same minute day after day, or will wake punctually at an unusual hour determined upon overnight. How can this knowledge of an hour (more accurate often than anything the waking consciousness shows) be possible without mental activity during the interval?
>
> Such are what we may call the classical reasons for admitting that the mind is active even when the person afterwards ignores the fact. Of late years, or rather, one may say, of late months, they have been reinforced by a lot of curious observations made on hysterical and hypnotic subjects, which prove the existence of a highly developed consciousness in places where it has hitherto not been suspected at all . . . That at least four different and in a certain sense rival observers should agree in the same conclusion justifies us in accepting the conclusion as true [JAME2: 131-132].

While some may take issue with James on whether or not we *always* "catch ourselves in the middle of a dream" when we are suddenly awakened, the other facts that he recites above deserve our serious consideration. It is one thing to say that we do not remember 'what happened' or, better put, 'what we perceived' while we were asleep; it is something altogether different to say 'the mind was inactive' during sleep. The former touches not upon the character of subjective time, while the latter may well. Is mind ever inactive? This is a difficult question in many ways because 'mind' is a supersensible object, not a sensuously observable object.

> The case of time-gaps[3], as the simplest, shall be taken first. And first of all, a word about time-gaps of which the consciousness may not be itself aware.
>
> On page 130 we saw that such time-gaps existed, and that they might be more numerous than is usually supposed. If the consciousness is not aware of them, it cannot feel them as interruptions. In the unconsciousness produced by nitrous oxide and other anæsthetics, in that of epilepsy and fainting, the broken edges of sentient life may meet and merge over the gap, much as the feelings of space of the opposite margins of the "blind spot" meet and merge over that objective interruption to the sensitiveness of the eye. Such unconscious as this, whatever it may be for the onlooking psychologist, is for itself unbroken. It *feels* unbroken; a waking day of it is sensibly a unit as long as that day lasts, in the sense in which the hours themselves are units, as having all their parts next each other with no intrusive alien substance in between. To expect the consciousness to feel the interruptions of its objective continuity as gaps would be like expecting the eye to feel a gap of silence because it does not hear, or the ear to feel a gap of darkness because it does not see. So much for the gaps that are unfelt.
>
> With felt gaps the case is different. On waking from sleep, we usually know that we have been unconscious, and we often have an accurate judgment of how long. The judgment here is certainly

[3] Bear in mind that here James does not mean 'time has gaps' but that our consciousness of 'events in external reality' has 'gaps in time.'

> an inference from sensible signs, and its ease is due to long practice in the particular field. The result of it, however, is that the consciousness is, *for itself*, not what it was in the former case, but interrupted and discontinuous, in the mere sense of words. But in the other sense of continuity, the sense of the parts being inwardly connected and belonging together because they are parts of a common whole, the consciousness remains sensibly continuous and one. What now is the common whole? The natural name for it is *myself, I,* or *me* [JAME2: 154-155].

What should catch our notice in these remarks is James' use of the word *feeling*. The basis of his thesis that consciousness is 'sensibly continuous' is that it 'feels continuous'; this is a point to which we will return later.

Freud also had an interest in the phenomenon of sleep, although this interest seems to have been secondary to his main interest in the phenomenon of dreams. The details of Freud's dream theory are not of particular interest to us in this treatise, but his review of some of the psychological findings and theories of sleep, such as they were in his day, is of interest. Freud reviews a number of findings and interpretations from other researchers, and a sampling of this is provided in the following. He does not hesitate to juxtapose quite different views and theories, many of which stand in flat contradiction, in terms of theoretical conclusions, from one another.

> It has been justly remarked that one of the chief peculiarities of dream-life makes its appearance even in the state of falling asleep, and may be defined as the sleep-heralding phenomenon. According to Schleiermacher[4] . . . the distinguishing characteristic of the waking state is the fact that its psychic activity occurs in the form of ideas rather than in that of images. But the dream thinks mainly in visual images, and it may be noted that with the approach of sleep the voluntary activities become impeded in proportion as involuntary representations make their appearance, the latter belonging entirely to the category of images. The incapacity for such ideational activities as we feel to be deliberately willed, and the emergence of visual images, which is regularly connected with this distraction – these are two constant characteristics of dreams, and on psychological analysis we are compelled to recognize them as essential characteristics of dream-life. As for the images themselves – the hypnogogic hallucinations – we have learned that even in their content they are identical with dream-images.
>
> Dreams, then, think preponderantly, but not exclusively, in visual images. They make use also of auditory images, and, to a lesser extent, of the other sensory impressions. Moreover, in dreams, as in the waking state, many things are simply thought or imagined (probably with the help of verbal conceptions). Characteristic of dreams, however, are only those elements of their contents which behave like images, that is, more closely resemble perceptions than mnemonic representations. Without entering in upon a discussion of the nature of hallucinations – a discussion familiar to every psychiatrist – we may say, with every well-informed authority, that the dream hallucinates – that is, it replaces thoughts by hallucinations. In this respect visual and acoustic impressions behave in the same way. It has been observed that the recollection of a succession of notes heard as we are falling asleep becomes transformed, when we have fallen asleep, into a hallucination of the same melody, to give place, each time we wake, to the fainter and qualitatively different representations of the memory, and resuming, each time we doze off again, its hallucinatory character [FREU11: 157-158].

Two remarks are in order here, justifying the interruption of the continuity of Freud's narrative. I think it likely that nearly every present-day psychologist and psychiatrist would point out that a

[4] Schleiermacher, Fr., *Psychologie*, L. George (ed.), Berlin, 1862, pg. 351.

great deal more has been learned about sleep- and dream- phenomena than was known in Freud's day, and in the nineteenth century psychology and psychiatry of his sources. Assertions such as those made above have to be based on either introspection or upon self-reporting by the subject (which is also based on introspection). Psychology has learned how unreliable this can be. So far as 'dream images' being "the sleep-heralding phenomenon" is concerned, I am personally not disposed to accept this idea at face value. The not-frequent occasions where I have observed what I would call 'the sleep-heralding phenomenon' does not match up with this. These occasions arise only when I am attempting to fight off sleep, when I, metaphorically speaking, drift across that fuzzy boundary between 'being awake' and 'falling asleep' but am able to briefly 'pull myself back' to 'the conscious side.' Here what I notice most is that it is my visual perception which first 'goes dark' while auditory perception, to an ever-lessening degree, remains. This is, of course, an introspection and does not at all merit being called an 'experiment' in any sense of that word. But it is my own experience in taking note of the onset of sleep, and the experience is quite different from the 'visual images as herald' theory described above.

The other comment I make here is that the distinction made above by the old-time researchers between 'thoughts' and 'hallucinations' is at best a vague distinction and calls upon a number of ontological presuppositions in making such a distinction. Modern psychology defines "hallucination" as "a perceptual experience with all the compelling subjective properties of a real sensory impression but without the normal physical stimulus for that sensory modality." Insofar as this goes, the modern definition is compatible with that of Freud and his predecessors. But modern psychology likes to draw a distinction between 'hallucination' and other 'false perceptions' that occur normally, like the images that often accompany the transition from waking to sleeping (hypnagogic) or those that occur when first awakening (hypnopompic).[5] The Critical definition of 'thinking' is "cognition through concepts," and if 'false perceptions' means imagination-driven cognition (of a fantasy), then there is no Critical distinction between 'thoughts' and 'hallucinations' as Freud describes them. Dreams thus seem to employ the same noetic capacities as does 'waking thought' insofar as apprehension and imagination are concerned.

But getting back to Freud's summary,

> The transformation of an idea into a hallucination is not the only departure of the dream from the more or less corresponding waking thought. From these images the dream creates a situation; it represents something as actually present; it dramatizes an idea, as Spitta[6] puts it. But the peculiar character of this dream-life is completely intelligible only if we admit that in dreaming we do not as

[5] Reber's *Dictionary*.
[6] Spitta, W., *Die Schlaf- und Traumsustände der menschlichen Seele*, 2nd ed., Freiburg i.B., 1892.

a rule . . . suppose ourselves to be thinking, but actually experiencing; that is, we accept the hallucination in perfectly good faith. The criticism that one has experienced nothing, but that one has merely been thinking in a peculiar manner – dreaming – occurs to us only on waking. It is this characteristic which distinguishes the genuine dream from the day-dream, which is never confused with reality.

The characteristics of the dream-life thus far considered have been summed up by Burdach[7] as follows: "As characteristic features of the dream we may state (a) that the subjective activity of our psyche appears as objective, inasmuch as our perceptive faculties apprehend the products of phantasy as though they were sensory activities . . . (b) that sleep abrogates our voluntary action; hence falling asleep involves a certain degree of passivity, . . . The images of sleep are conditioned by the relaxation of our powers of will.[8]

It now remains for us to account for the credulity of the mind in respect to the dream-hallucinations which are able to make their appearance only after the suspension of certain voluntary powers. Strümpell[9] asserts that in this respect the psyche behaves correctly and in conformity with its mechanism. The dream-elements are by no means mere representations, but true and actual experiences of the psyche, similar to those which come to the waking state by way of the senses. Whereas in the waking state the mind thinks and imagines by means of verbal images and language, in dreams it thinks and imagines in actual perceptual images. Dreams, moreover, reveal a spatial consciousness, inasmuch as in dreams, just as in the waking state, sensations and images are transposed into outer space. It must therefore be admitted that in dreams the mind preserves the same attitude with respect of images and perceptions as in the waking state. And if it forms erroneous conclusions in respect of these images and perceptions, this is due to the fact that in sleep it is deprived of that criterion which alone can distinguish between sensory perceptions emanating from within and those coming from without. It is unable to subject its images to those tests which alone can prove their objective reality. Further, it neglects to differentiate between those images which can be exchanged at will and those in respect of which there is no free choice. It errs because it cannot apply the law of causality to the content of its dreams. In brief, its alienation from the outer world is the very reason for its belief in its subjective dream-world [FREU11: 158].

We must separate the wheat from the chaff in these characterizations. The non-Critical ontology of 'reality' and the positivist-empiricist presuppositions contained here are obvious and we will not belabor them. One key point here is that dream-appearances are no different from those we experience in the 'waking state' and that 'experience in the dream-state' differs from 'experience in the waking state' only insofar as dreamer seems to apply no ratio-expression we might call a 'hey, wait a minute' when his dreams involve some fantastic representation (such as the ability to fly or showing up for work in one's pajamas). This can be all that is meant in saying 'the mind cannot apply the law of causality to the content of its dreams' because, as we all know for ourselves, dream-appearances exhibit not only a presentation in space but also a sequence in time. If dreaming is merely "thinking in a peculiar manner," this characterization merely emphasizes the comment made above.

One characterization of dreams common in Freud's day was that their fantastic 'nature' was the result of sleep 'shutting out the outside world,' and that our ability to understand that we were

[7] Burdach, *Die Physiologie als Erfahrungswissenschaft*, vol. III, pg. 476, 1830.
[8] From the Critical standpoint, the term 'will' has objective validity only with regard to acts of the synthesis of appetition (Chapter 20). The phrase 'relaxation of our powers of will' as it is used here by Freud has no objectively valid meaning in our theory.
[9] Strümpell, L., *Die Natur und Entstehung der Träume*, Leipzig, 1877.

dreaming is restored upon awakening by our 're-contact' with that 'outside world.' Freud does not agree that this is a complete explanation, and he again cites Burdach to support this point.

> If the turning-away from the outer world is accepted as the decisive cause of the most conspicuous characteristics of our dreams, it will be worth our while to consider certain subtle observations of Burdach's which will throw some light on the relation of the sleeping psyche to the outer world, and at the same time serve to prevent our over-estimating the importance of the above deductions. "Sleep," says Burdach, "results only under the condition that the mind is not excited by sensory stimuli . . . yet it is not so much a lack of sensory stimuli that conditions sleep as a lack of interest in them; some sensory impressions are even necessary in so far as they serve to calm the mind; thus the miller can fall asleep only when he hears the clatter of his mill, and he who finds it necessary, as a matter of precaution, to burn a light at night cannot fall asleep in the dark."
>
> "During sleep the psyche isolates itself from the outer world, and withdraws from the periphery . . . Nevertheless, the connection is not entirely broken; if one did not hear and feel during sleep, but only after waking, one would assuredly never be awakened at all. The continuation of sensation is even more plainly shown by the fact that we are not always awakened by the mere force of sensory impression, but by its relation to the psyche. An indifferent word does not rouse the sleeper, but if called by name he wakes . . . so that even in sleep the psyche discriminates between sensations . . . Hence one may even be awakened by the obliteration of a sensory stimulus, if this is related to anything of imagined importance. Thus one man wakes when the nightlight is extinguished, and the miller when his mill comes to a standstill; that is, waking is due to the cessation of a sensory activity, and this presupposes that the activity has been perceived, but has not disturbed the mind, its effect being indifferent, or actually reassuring" [FREU11: 158-159].

Although we can be plainly skeptical about some of the examples and the nineteenth century interpretations applied to them, it seems as clear and sure as anything in experience that receptivity can intrude upon the sleeping state and rouse us to wakefulness. The ringing telephone, a touch on the shoulder, or even a strange sound in the night can awaken a sleeper. It is true enough that the degree of sensation involved in this varies during the night. Neuroscience has shown that there are measurable phases of sleep during which the ease of arousal is measurably different. It is likewise known that alcohol and drugs have measurable effects on the physiological characteristics of the 'sleeping state,' as anyone who has tried to wake a drunk well knows. Freud has a great deal more to say, but for our purposes the main points have already been raised: there is psychological evidence that the processes of sensibility are not inactive during sleep, and that in at least the 'dream-state' perception is active, even if the ability to remember a dream appears to be impaired or even disabled.

These observations and theories, made in the youth of psychology, have the value attending many early essays at the dawn of a new science. The early psychologists' first task, of necessity, had to be one of identification and characterization of the phenomena of their topic. In all sciences it is the case that, as theory develops, the earliest observations of phenomena tend to be less and less well-regarded as scientists more and more favor those topics that theory, which brings structure to the doctrine, better deals with. Yet, barring their exclusion from the domain of the topic, these early observations and facts yet remain pertinent to the science and so it has been worth our while to spend time on these here. But let us now turn to our more modern findings of fact, particularly those gained by modern neuroscience.

Neuroscience cannot observe the supersensible *nous* but it can and does make observations and experiments on *soma*. By our applied metaphysic of the data of the senses (Chapter 6) and, more particularly, by the principle of emergent properties, we already know that whatever signaling activities we discover in brain, to these we must ascribe a reciprocal activity in *nous*. With regard to the phenomenon of sleep, our scientific picture of sleep as we left the twentieth century is more complete than that with which science entered it, and it does confirm some of findings reported above by the nineteenth century researchers, and refutes others.

Electroencephalograph (EEG) studies show that sleep is divided into two phases, rapid-eye-movement (REM) and non-REM sleep (also known as slow-wave sleep). Non-REM sleep is further divided into four stages, labeled 1 through 4 for the order in which they occur after the subject first falls asleep. Sleep follows a sequence from waking-state EEG activity to non-REM stages 1, 2, 3, and 4. This is followed by a reversal leading from stage 4 back to stage 1. The first REM sleep follows stage 1 and then the cycle repeats. In adults the complete cycle takes 90 minutes to two hours and repeats four or five times during sleep.

Although it was initially thought that dreaming takes place only during REM sleep, studies have shown that this is not true and that dreaming also occurs during non-REM sleep.

> Although REM sleep is the phase from which dreams may be most reliably elicited, REM sleep is not necessary for dreaming . . . Reports of non-REM dreams tend to be shorter, less vivid, less emotional, and more coherent than reports of REM dreams. But there are no qualitative differences between REM and non-REM reports of the same length. Thus, the major difference is that REM dreams tend to be longer than non-REM dreams.
>
> REM sleep is not sufficient for dreaming, which varies with cognitive abilities as well as sleep stages. Even though children have abundant REM sleep, they rarely report thematically organized dreams before ages 7-9 years; appearance of organized dreams is correlated with the development of visuospatial skills. Dreaming may be absent in a variety of patients with neurological damage who nevertheless show REM sleep.[10]

Modern studies confirm that dreams are experienced as 'real' and that we are able to distinguish between 'real' and 'imagined' mental images only when we are awake. Measured durations of dreams, determined by correlating self-reported factors with EEG measurements of the length of REM sleep, show that there is no distortion or 'compression' of the 'passage of time' during dreaming.

> Despite many popular anecdotes to the contrary, the passage of time in dreams is not compressed. On the assumption that it would take more words to describe a long dream than a short one, Dement counted the number of words in dream reports and compared these to the length of the REM

[10] Allan Rechtschaffen and Jerome Siegel, "Sleep and dreaming," in [KANDa: 936-947].

> episodes. The length of dream narratives showed a highly positive correlation with the duration of REM sleep. In another experiment in the same series, Dement awakened subjects either 5 or 15 minutes after the onset of REM sleep and asked them to specify whether they had dreamt a short or long time based upon the apparent duration of whatever dream material they could recall. A correct choice was made in 83% of instances.
>
> Although vivid dreaming occurs primarily in REM sleep, mental activity also occurs during slow-wave sleep. In general, mentation during slow-wave sleep is more poorly recalled, less vivid and visual, more conceptual and plausible, under greater volitional control, less emotional, and more pleasant. An important exception is that most episodes of sleep terror nightmares occur during stages 3 and 4 of slow-wave sleep . . . However, as is typical of mental activity during slow-wave sleep, such episodes are not accompanied by full dream narratives; rather, a single oppressive situation is recalled, such as being locked up in a tomb.[11]

Contrary to previous opinions, it appears that dreams are not rare. The studies show that healthy individuals dreams in regular cycles several times each night. Table 21.4.1 below shows some of the characteristics of dreams as self-reported by experimental subjects. Contrary to views held in Freud's time, 'thinking' is *not* absent in dream activity, although 'thinking content' is significantly lower in REM sleep than in non-REM sleep. One hypothesis that has been offered is that REM sleep might play a role in 'setting' longer-term memories of episodic events. There is some neurological evidence to support this, but the function or biological benefit provided by sleep remains unexplained to this day.

Empirically, then, sleep is characterized by dream events and, apparently, periods in which no dreaming seems to take place. Where dreams are present the representative content appears no

Table 21.4.1. Self-reported Characteristics of Mental Activity During REM and non-REM Sleep

Characteristic	Sleep Stage: Slow-wave 3 and 4	Sleep Stage: Ascending 2	Sleep Stage: REM
Features present (percent positive response)			
Dreaming content	51	51	82
Thinking content	19	23	5
Emotion felt by self	28	29	50
Visual	73	62	90
Physical movement of self	33	38	67
Only one other character	62	50	34
Shift in scene	28	38	63
Recall makes sense to dreamer in terms of recent experiences	69	75	48
Mean self-rating of dream characteristics (0 = Low, 5 = High)			
Anxiety	0.71	1.00	1.19
Violence/hostility	0.12	0.59	0.71
Distortion	1.12	0.41	1.68

Adapted from Kelly [KAND: 798].

[11] Dennis D. Kelly, "Sleep and dreaming," in [KAND: 792-804].

different insofar as appearances are concerned than waking-state representations. Therefore, to the extent that 'time' seems 'continuous' in the waking state, it is equally 'continuous' in dreaming. As for sleep where dream activity has not been detected, this implies a lack of experienced representation, for which mere suspension of judgment is sufficient. This means that here, so far as modeling subjective time is concerned, the question of 'continuity' is moot. We cannot talk about what is necessary for the possibility of experience where no experience is to be found.

Subjective Time is Objectively Continuous: The Mathematical Aspect

In a one-dimensional model of subjective time 'continuity' is a built-in characteristic. The sequence of kinetics (a, b), (b, c), (c, d), etc. in intuition is merged as one moment in time is the direct cover of the next. Even though such a mathematical representation when written down appears discrete, *experience* requires perception and perception is indissolvably bound to moments in time. To speculate on ontological *extensive* magnitudes (aggregate of 'simples') 'between' moments in time has no objective validity because there is no possibility whatsoever that such *noumena* could be objects of any possible experience, and the *Existenz* of such *noumena* is in no way necessary for the possibility of experience. When we say that a moment in time 'grows out of the previous moment,' all we are saying is that temporally-ordered *change* is a perceptible phenomenon. It is true enough, as James argued, that we notice within the 'stream of thought' two characteristics, the 'transitive' and the 'substantive' parts. This, however, we can now see as the consequence of the inference of ideation. Not every intuition is made a concept by the synthesis of re-cognition, and those which are not are suited to the role of the 'transitive' part of James' model because these objective perceptions are not distinct *cognitions*.

However, in a multi-dimensional timescape the question of 'continuity' is not so straight-forward. Referring back to figure 21.3.1, this model contains intuitions (e.g. at s_j and s_k) that do not form a chain and for which neither is a cover for the other. So far as *objective* perception is concerned, we cannot say there is continuity between these objective perceptions and, indeed, we must say that in the subjective timescape this 'discontinuity' is characteristic of intuitions.

But this is not the entire story. At every moment in subjective time there are also affective perceptions. These, by their very definition, are representations that are not and do not become part of any objective representation. *Affective perceptions form a single chain of ordered pairs*. Continuity in inner sense is continuity in subjective time by means of continuity in affective perceptions. This is something we might have anticipated from our previous discussion of the synthesis in continuity in the aesthetical Idea, which we discussed in Chapter 16 (§6.2). This is the synthesis of Quality in the unity between reflective judgment and noetic *Kraft* of adaptive *psyche*. Affectivity, seen in this way, serves as a kind of sensuous substratum for continuity, in which all 'parts of time' are merely limitations and by which our model of subjective time, seen as object of the idea, *is* continuous *as an Object*.

§ 4.5 Free Time

'Free time' is the name I use in this treatise for the idea of the model (Nature) of time in its application to modeling and understanding the processes of Reason. As an Object, free time is and can be nothing else than a functional *noumenon* with nothing else than merely practical objective validity, this validity being grounded in the necessary unity of the mental anatomy and mental physiology of *nous*. Pure Reason is not bound to determination by the pure synthesis of inner sense (pure intuition of time), and indeed we say in Chapter 20 that ratio-expression affects the structuring of subjective time through regulated re-introduction of concepts back into the synthesis of apprehension.

The pure practical aim of the central process of equilibration is nothing less than the Ideal of absolute perfection in sensibility and reflective judgment (aesthetical perfection), cognition (logical perfection) and appetition in accordance with universal practical laws (practical perfection). The transcendental question of free time is therefore: What is necessary for the possibility of acting to perfect in practical Reason's synthesis of the manifold of rules and the synthesis of appetition? Now, pure practical Reason has no immediate interest in either cognition or sensibility. However, it is clear that the forms of sensibility and of the manifold of concepts affect the achievement of aesthetical and logical perfection, and that progress in these perfections goes hand-in-glove with practical perfection. Practical Reason affects sensibility in one way through its determination of appetites (which exercise Reason's 'veto power' over motoregulatory expression and, thereby, influence kinæsthetic feedback in receptivity). This, however, is an act of immediate judgment for practical Reason since it requires no immediate cooperation from the other faculties of judgmentation in *nous*. It is otherwise with Reason's acts of ratio-expression because these acts affect sensibility – and therefore the synthesis of subjective time – through spontaneity.

Motivation is the accommodation of perceptions, and we have seen (Chapter 20) that the successful assimilation of acts of reflective judgment in the practical manifold of rules requires this act of accommodation. In this context ratio-expression is the intellectual expression of appetite aimed at achieving this accommodation. Now, we have seen (Chapter 17) that the topological structuring of subjective space obtains its sensuous *materia circa quam* from the actions of motoregulatory expression. It will then, perhaps, come as no surprise that the ordering structure of subjective time obtains *materia circa quam* in sensibility through noetic actions regulated by ratio-expression. From this it follows that the logical essence of free time is grounded in the development of a synthesized order structure in subjective time in such a way that Reason's Ideal of practical perfection is possible.

Appetites are representations of practical Reason which, in the appearances of their consequences, have the 'forward-looking' character we commonly call our expectations of satisfactions. In the evolution of mental development this forward-looking character eventually appears as the phenomena of the ability *to plan, to foresee outcomes, and to 'time' the realization of our actions*. Indeed, the *conceptualization of practical tenets* is the cognitive outcome of this practical process. The most common usage of the word *intelligence* carries this precise connotation. Simple anticipation, e.g. the imaginative reproduction of the intuition of *Unsache*-thing {1, 2, 3} from our previous example, is sufficient to make possible the relatively simple circular reactions of the early stages of sensorimotor intelligence. But the development of what Piaget calls "operations" requires something more than this. It requires that it be possible, from the transcendental schemata that provide homogeneity between concepts and intuitions, to construct a manifold of concepts that supports the formation of teleological concepts – i.e. concepts by which the Organized Being can 'look backward' from the concept of a desired outcome to concepts of means of obtaining this 'future' outcome.

Because such 'planning concepts' are inherently *a priori* in their cognitive logical essence (that is, they are concepts of 'things that haven't happened yet'), they are necessarily the products of inferences of ideation, of induction, or of analogy in reflective judgment. They are *ideas,* placed in the manifold of concepts by determining judgment but originating from reflective judgments of formal expedience. Obviously concepts of objective time are necessary for the synthesis of such ideas (which is why pre-operational children and children in the developmental stage of concrete operations, where ideas of objective time are not yet well-structured, do not exhibit 'good planning abilities'). From this it follows that: in the ability to construct well-equilibrated ideas of objective time in cognition we find the characteristic of thinking in which the logical essence of free time finds cognitive exhibition. We are thus led to consider what is necessary for the perfection of the idea of objective time in cognition.

Now, the synthesis of the pure intuition of time, as the synthesis of order structure, lacks only one characteristic necessary for the perfection of cognitive objective time. This characteristic is *reversibility*. The partial orderings in subjective time are antisymmetric, which means that these representations 'run in only one direction.' Reversibility in time means that for the subjective partial ordering (a, b) one can set in correspondence a reversed ordering (b, a). Such a reversal is impossible for the pure intuition of time acting alone. But it is otherwise in the free play of imagination and determining judgment because the categories of understanding are rules of judgment by which time-determinations are established in determinate combinations of concepts. Reversibility in ideas of objective time is made possible through the employment of the process of determining judgment acting through the synthesis of reproduction in imagination.

Therefore, we can state the logical essence of free time thusly: **Free time is the ideal of Reason's capacity for ratio-expression such that, through expression of the transcendental Ideas of speculative Reason, it is possible to synthesize in intuition ordered pairs that constitute temporal inverses of previous partial orders**. This synthesis must obviously be a synthesis of comprehension rather than merely a synthesis of apprehension. The latter supplies the original ordered pairs for which comprehension produces the inverses. It is to be emphasized that the resulting order structure is an order structure in *the concept of objective time* and is not a new capacity of the pure intuition of time.

In mathematics such a reversible order structure is called a *lattice*. The branch points in subjective time that we illustrated earlier are *joins* in time-reversed connections. In mathematics this is denoted by calling them 'meets.' The joins in subjective time are branch points in time-reversed connections. The essential character of free time is the ability, through ratio-expression, for practical Reason to stimulate representations in the synthesis of comprehension that lead to time-reversed combinations of concepts by determining judgment. In the absence of an applied metaphysic of mental physics, it is mere speculation to postulate specific reasonings for such a formative process. Nonetheless, the grounds for the mathematical possibility can be simply illustrated as shown in Figure 21.4.5. Here the open circles denote intuitions arising from concept reproduction. The solid circles denote intuitions of apprehension, i.e. products of receptivity from

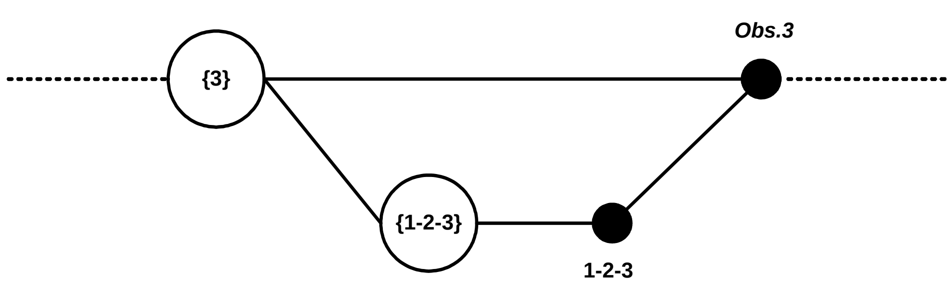

Figure 21.4.5: A possible synthesis in comprehension satisfying the condition for the possibility of lattice structuring in determining judgment. Open circles represent intuitions reproduced via the synthesis of imagination. Filled circle 1-2-3 represents a sensuous intuition of apprehension due to the actualization of scheme 1-2-3. *Obs.3* represents the sensuous intuition of the object {3}.

the *materia ex qua* of the senses. {3} denotes the intuition of a *Sache*-thing regarded as an object of Desire. {1-2-3} denotes the intuition of an *Unsache*-thing, namely a scheme for which {3} is the anticipated outcome. Solid circle 1-2-3 represents the intuition of apprehension resulting from motoregulatory expression of the scheme. Finally, *Obs.3* is the intuition of apprehension following the successful application of the scheme.

In figure 21.4.5 we have assumed that there has been a prior conceptualization giving rise to the two intuitions denoted by open circles. Thus, we have already at least the formation of a Type I interaction structure because the concept of {3} is the concept of a Piagetian *Obs.O* (the intuition of which is denoted *Obs.3* in the figure) and the concept of {1-2-3} is the concept of a Piagetian *Obs.S*. No representation of an objective time (as an object) is presented in Figure 21.4.5, and if we presume that the Subject is still in the sensorimotor stages of mental development no clear and distinct concept of *noumenal* time is yet to be expected. However, the kinetic ({3}, {1-2-3}) presages the sort of reversal in time-ordering that is a necessary first condition for the eventual ability to conceptualize a reversed sequence {3}-{2}-{1}.

This condition, while necessary, is not sufficient for constructing an objective time concept in which this reversibility is a characteristic. For example, the figure implies no affective condition that requires {3}-{1-2-3} to be conceptualized as an *Unsache*-thing (the interaction comes to the condition for equilibrium at first trial in the figure). For such a conceptualization to occur, there must be some "resistance" (Chapter 9 §2.2) encountered in the execution of the scheme. This situation is the sort we could expect to find empirically in stage 4 of sensorimotor intelligence, the developmental stage when schemes become "mobile." In discussing this stage, Piaget writes:

> The conclusion to which we are led is that the coördination of means to ends always involves a reciprocal assimilation of the present schemes as well as a correlative putting into relationships of the objects subsumed by these schemes. In the simplest cases this double assimilation is almost equivalent to a fusion and so calls to mind that which accounts for the coördination of the primary schemes. In other cases it can also remain truly reciprocal and give rise in that way to symmetrical series . . . In order to understand this diversity we should emphasize a fact to which we have already referred and which will assume great importance in the rest of our analysis: This is the functional analogy of the schemes of this stage (and of the following stages) with concepts, of their assimilation with judgments and of their coördinations with logical operations or reasonings.
>
> From the point of view of *assimilation*, two complementary aspects characterize the schemes of which we have just spoken, when we compare them to the secondary schemes of the third stage from which they nonetheless derive: they are more mobile and consequently more generic. True, the secondary schemes encroaches upon all the characteristics of the "mobile" schemes peculiar to this stage, but in a form to some degree more condensed (because undifferentiated) and consequently more rigid. This secondary scheme is a complete totality of intercoordinated movements and functions every time the child perceives the objective in connection with which the scheme was formed, or analogous objectives . . . But, if one examines this closely, one notices that certain essential differences are in opposition to the simple secondary scheme (that of the third stage), the same scheme having become "mobile" during the present stage. At first the relations between

> objects, relations already utilized by the secondary scheme, are given just as they are in the midst of the latter without the child's having elaborated them intentionally, whereas the relations due to coördination of the "mobile" schemes have really been constructed by the subject . . . They intercoördinate and consequently dissociate to regroup in a new way, the relations which they involve, each in itself, becoming capable of being extracted from their respective totalities to give rise to various combinations. Now these various novelties have combined solidarity. In becoming "mobile" – that is to say, fit for new coördinations and syntheses – the secondary schemes have become detached from their usual contents to apply themselves to a growing number of objects. From particular schemes with special or peculiar contents they accordingly become generic schemes with multiple contents [PIAG1: 236-238].

These newly "mobile" schemes become more elaborated during sensorimotor stage 5, when the child actively experiments with the schemes themselves. Stage 5 is the stage of the 'tertiary circular reaction' and of "the discovery of new means through active experimentation" [PIAG1: 263-330]. In this remarkable stage, the child 'plays with' his repertoire of sensorimotor schemes, his interest and attention apparently being given over, for the first time, to the deliberate accommodation of schemes, apparently simply for the sake of accommodating them.

> With the advent of tertiary circular reactions, accommodation becomes an end in itself which certainly protracts the earlier assimilations . . . but which precedes new assimilations and so intentionally differentiates the schemes from which it sprang . . . Henceforth there exists interest in the new as such . . . Accommodation to experience and deductive assimilation henceforth alternate in a movement whose rhythm can vary but whose cyclical character attests to an increasingly close correlation between the two terms . . . groping accommodation goes in quest of new schemes capable of being coordinated with the old ones. The "discovery of new means through active experimentation" thus marks the beginning of a union of experience and assimilatory activity, a union which "invention through mental combination"[12] will consecrate by raising it to the rank of interdependence [PIAG1: 325].

Although we can very well regard the 'reversed' kinetics in subjective time as a mere by-product of these sensorimotor activities, the key thing to note is the volitional appearance of the stage 5 actions. Nothing 'external' forces the child to behave in stage 5 fashion. It is equally clear that actions in this stage have progressed past simple Type I interactions to incorporate a non-observable factor, i.e. these are Type II interactions in equilibration. Although nothing in stage 5 (or, for that matter, stage 6) gives evidence of the form of *Existenz* of child's concept of objective time, it is easy to see how experience gained through these behaviors is propaedeutic to the latter accommodations of manifold concepts of objective time.

Now, since the behaviors characteristic of these stages lead to concept formation, rather than being the consequence of concepts previously formed that give practical necessitation for the actions, we can only lay the ground for these behaviors with pure practical Reason. Because Type II interactions involve Piagetian coordinations (which are not observables), there is no other source in motivation for these coordinations (*Coord.S* and *Coord.O*) other than the manifold of

[12] Stage 6 of sensorimotor development.

concepts. But practical Reason knows no concepts of understanding and so if such concepts are brought into the synthesis of sensibility (and thence into reflective judgment as desires and desiration), we can only lay the possibility of the summons of such concepts into conscious representation to a *form* of ratio-expression through speculative Reason and its transcendental regulative principles. The Object of the idea of such a form is that which here is called free time.

Action series in the manifold of concepts are connected under the notion of causality & dependency, which is the notion corresponding to the *modus* of succession in time. To *think* the series *from* a teleological objective (e.g. {3}) *to* a scheme of means (e.g. {1-2-3}) in its first occurrence requires of determining judgment a synthesis *a parte priori* (that is, a synthesis 'ascending' the series of concepts) such that {3} is brought to sensibility first, followed by the imaginative reproduction of {1-2-3}. But here the following consideration comes into play. When the concept {3} is first made under the notion of substance & accident, its Relation to *Unsache*-thing {1-2-3} is the notion of community. This is because {3} is obtained by abstraction from the concept {1-2-3}, and in this the *modus* of subjective time is coexistence. Therefore, in order for the synthesis in comprehension to produce succession in subjective time, a transformation is necessary (because connection of the manifold under community does not in itself necessarily imply connection under causality & dependency). This transformation must be one that produces a connection (through the free play of imagination and determining judgment) under the notion of causality & dependency, but in this case the character of the Relation in terms of its concept-content is that of a theoretical *nexus finalis* (a teleological postulate) rather than *nexus effectus* (efficient cause). This transformation is an anasynthesis, and its possibility can only be laid to the causality of freedom since receptivity in appearances can bring no such connection.

The starting point of such an anasynthesis is the regulative principle of Quantity in the empirical-theoretical perspective. This is the Idea of *entis realissimi* from the theoretical Standpoint: the synthesis of all possible predicates in one Object. The connection in the manifold of concepts {3}-{1-2-3} is a 'predicate' in the logical structure of the manifold of concepts. But this particular prior 'predicate' is made under the notion of community. The predication in thinking of {3}-{1-2-3} is likewise possible under the notion of causality & dependency, but the ground of this possibility, because it can arise only from spontaneity, subsists in the regulative Idea of *entis realissimi*. Contrast this with our earlier example {1}-{2-3}. In that case, Relation in the manifold of concepts was already Relation under causality & dependency, hence a predicate already in the form of succession in subjective time. For this the regulative principle of the cosmological Idea of Relation (absolute completeness in the beginning of an appearance generally) is sufficient for ascending the series in the manifold of concepts to reach {1}, and the remaining concept {2-3} needs no additional synthesis of comprehension in order to follow as a kinetic ({1}, {2-3}) in the synthesis of apprehension.

What we see empirically in the behavior of the stage 5 child can be regarded as practical Reason acting to perfect its own manifold of rules (construction of higher practical tenets). But this requires the employment of determining judgment in a specific manner – the aforementioned process of anasynthesis – in order for thinking to produce the motivation (accommodation of perception) in the presentations of reflective judgment that are necessary in order that this practical acting-to-perfect-the-manifold-of-rules should be able to succeed. To make a theory of this employment we necessarily must represent this action in temporal (and spatial) terms, since that is the Nature of human understanding. To have such a representation we must regard the form of ratio-expression in temporal terms, hence our *practical need* to posit free time as the form of regulative employment of judgmentation in general. Transcendental free time is therefore the form of practical 'regulation of the regulations' of speculative Reason. It is, so to speak, **the logical form of the causality of freedom in action insofar as this action is ratio-expression through speculative Reason**.

In logical perspective, free time is a mere logical sequencing of acts of speculative Reason in the service of structuring ever more universal practical laws in the manifold of rules. We do not experience the practical acts *per se,* but we do experience the eventual consequences of these acts, and in appearances they are exhibited by the gradual construction of mobile schemes. However, the concept that a scheme *is* 'mobile' is a concept of coordination, and herein we have the earliest formation of the concept of *objective* time. Its detailed development in a theory of the Nature of free time is scientific theorizing that must always find its base in necessity for the possibility of these outcomes. From what we have learned in this treatise, there are a few observations we can make as to its character. The logical structure of free time makes possible the character of a *nexus finalis* in reasoning. This implies that lattice structure in the manifold of rules is expedient for the categorical imperative, and this implication is congruent with the Modality of will in choice. Therefore appetites in ratio-expression apply the regulative transcendental Ideas according to some optimization criterion favoring a lattice structure in the manifold of rules.

Free time is not bound to determination by subjective time. The matter of composition, for which free time is the form of *nexus,* subsists in the specific acts of the ratio-expression of the regulative transcendental Ideas (of Rational Psychology, Rational Cosmology, and Rational Theology) in speculative Reason. Beyond this understanding of free time, this treatise may not go without the aid of an applied metaphysic of mental physics.